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A SPECIAL BOUNDED SEQUENTIAL PROCEDURE

CPT ROBERT G. GORRIE

22 APRIL 1983

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The concept of item by item sequential sampling has been developed from the Wald sequential probability ration (SPR) test. The SPR plan for one-sided variables inspection (a plan for testing the mean $\bar{X}'$ of a normal distribution with known variance of σ'^2) is characterised by two parallel lines on a plt of the cumulative sum of observations T(n) versus accumulated sample size n. The operation of the plan is described by analyzing the squence T(n). For the case where small values of $\bar{X}'$ are preferred, $T(1) = X_1$ is the

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The number of observations, n, required for defining a course of action is random variable, often characterized by large values of n before a decision is made. Truncation rules are required to prevent unacceptable sample sizes even though convergence is guaranteed. This truncation increases the associated producer's and consumer's risk.

Herein a sequential non-probability ratio (SNPR) plan is examined which is characterized by intersecting lines and thus an implicit truncation property. A heuristic algorithm to define the parameters of the plan is introduced. The algorithm produces an SNPR plan which provides the same levels of protection as the non-truncated SPR plan.

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The Pennsylvania State University

The Graduate School

Department of Industrial and Management Systems Engineering

A Special Bounded Sequential Procedure

A Thesis in

Industrial Engineering and Operations Research

by

Robert G. Gorrie

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 1983

A

Robert G. Gorrie

The signatories below indicate that they have read and approved the thesis of Robert G. Gorrie.

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ABSTRACT

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ACKNOWLEDGMENTS

I would like to acknowledge the help and guidance of Richard D. Guild in the organization and development of this thesis. I would also like to acknowledge the inspiration provided by my father, George H. Gorrie, who showed me the real meaning of courage.

CHAPTER I

INTRODUCTION

Problem Statement

Abraham Wald developed a sequential probability ratio (SPR) test in 1943 to test statistical hypotheses. Initially used in sequential acceptance sampling by attributes, it has since been adapted to variables inspection. A unique feature of Wald's SPR test is that the number of observations required by the procedure is not determined in advance of the experiment. Its ability to quickly detect good or bad quality lots makes it superior to other acceptance sampling procedures. It can materially reduce the required amount of inspection by 50% on the average when compared to the single sampling plan.

While it has been shown that the probability equals 1 that the procedure eventually terminates [9], there is no effective upper limit on the number of items to be inspected. If lots are mediocre in quality, the number of units required to be inspected can be quite large. This problem is compensated for through the use of truncation rules which prohibit sampling beyond a certain number of units by forcing a decision at some a-priori fixed integer. If the procedure is truncated prematurely, its ability to discriminate between good and bad lots is weakened. If the procedure is truncated too late, its ability to discriminate is only marginally reduced but it loses its advantage of reducing the number required for inspection.

This study provides, through a procedure proposed by Guild [5], a sequential non-probability ratio (SNPR) test which is bounded for the case of variables inspection. The specific objectives of the

thesis are:

1. Development of a heuristic procedure which specifies the arguments of an SNPR procedure for a given set of parameters.
2. Comparison of SPR and SNPR procedures.
3. Development of a computer code for the heuristic and SNPR procedure.
4. Generation of a select set of SNPR test plans.

Introductory Remarks

The purpose of an acceptance sampling plan is to define a course of action; i.e., acceptance or rejection of a particular lot of given quality. If all lots are of the same quality, the plan will indicate acceptance of some lots and rejection of others, and the accepted lots will be no better than the rejected ones. If the lots differ in quality, the plan will accept good lots more frequently than it will bad lots [1].

Usually a plan is designed so that material considered to be of good quality will have a low probability of being rejected (producer's risk) while material considered to be of bad quality will have a low probability of being accepted (consumer's risk). The grade of material considered good is called the "Acceptable Quality Level" (AQL) and the probability of rejecting material of grade AQL or better is designated by α. Similarly, the grade of material below which rejection takes place is referred to as the "Lot Tolerence Percent Defective" (LTPD) and the probability of accepting material of grade LTPD is designated as β.

The precise relationship between lot quality and the probability of acceptance (P_a) of any given lot is shown by the operating characteristic (OC) curve of the particular sampling plan. The curve graphically illustrates the plan's ability to discriminate between different quality lots with fixed σ'. The pairs ($1-\alpha$, $\bar{X}'_{AQL}$) and (β, $\bar{X}'_{LTPD}$), where $\bar{X}'_{AQL}$ is the mean locating a percent defective equal to a desired AQL and $\bar{X}'_{LTPD}$ is the mean locating a percent defective to a desired lot tolerance LTPD, specify two points on the curve as shown in Figure 1. The point on the curve where P_a equals .5 locates a mean known as the indifference point ($\bar{X}'_a$). At this point, there is no particular preference for acceptance or rejection of the lot ($\alpha=\beta$). The curve shown in Figure 1 describes a plan where low values of $\bar{X}'$ are desirable. Besides showing lot quality relationships, the OC curve is also useful in evaluating alternate sampling plans.

The Chapters to Follow

Chapter II briefly examines a single sampling plan for variables inspection and then presents the development of Wald's SPR test for variables inspection. Also discussed are the methods used to calculate the OC curve and average sample number (ASN) curve of the SPR plan. Finally various truncation rules and their effect on the plan's discriminatory power are presented. The SNPR procedure and a heuristic method of estimating the procedures arguments are presented in Chapter III followed by a comparison of SPR and SNPR procedures in Chapter IV.

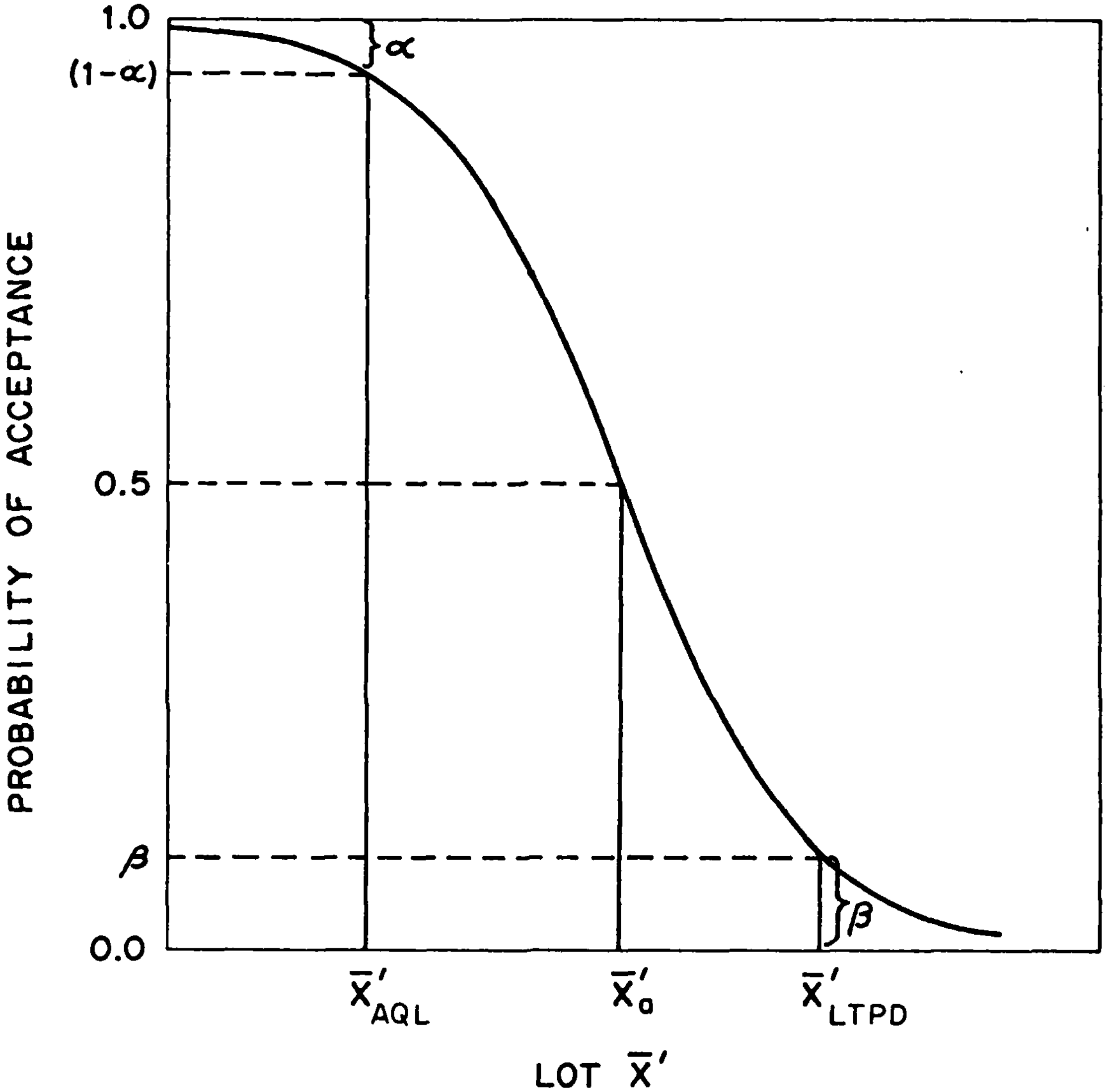

Figure 1. Operating Characteristic Curve

Throughout the thesis a sample problem is used to illustrate procedures and concepts. The problem requires the development of an acceptance sampling plan for variables inspection in which low values of $\bar{X}'$ are desirable. The following are the problem parameters:

$\bar{X}'_{AQL} = 200$ $\qquad \alpha = .05 \qquad \sigma' = 30$

$\bar{X}'_{LTPD} = 225$ $\qquad \beta = .10$

CHAPTER II

HISTORICAL PERSPECTIVE

Single Sampling Plan

A single sampling plan for variables inspection is relatively simple to design. Given $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, α, β and σ', a sample size N and an acceptance limit $\bar{X}_a$ are determined to establish the plan. The derivation of the plan is as follows. If a lot actually has a mean grade of AQL quality, means of samples of N from the lot will have a normal distribution with mean $\bar{X}'_{AQL}$ and a standard deviation of $\sigma'/\sqrt{N}$. Therefore, if α is the probability of rejecting a lot with mean $\bar{X}'_{AQL}$, then

$$\frac{\bar{X}_a - \bar{X}'_{AQL}}{\sigma'/\sqrt{N}} = Z_\alpha \qquad (1)$$

Similarly, a lot having a mean grade of $\bar{X}'_{LTPD}$ and a probability acceptance β requires

$$\frac{\bar{X}_a - \bar{X}'_{LTPD}}{\sigma'/\sqrt{N}} = -Z_\beta \qquad (2)$$

When $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, α, β and σ' are defined, we have two equations in $\bar{X}_a$ and N. The simultaneous solution of which yields values of $\bar{X}_a$ and N [1].

Using the problem defined in Chapter I, $\bar{X}_a$ is computed to be 214.05 and N to be 13. Figure 2 shows the OC curve for the plan.

Sequential Probability Ratio Sampling Plan

The SPR plan for variables [1] is one which requires the sequential observation of a variable X which is normally distributed with mean $\bar{X}'$ and standard deviation σ'. The decision variable is defined as:

$$T(n) = \sum_{i=1}^{n} X_i \qquad (3)$$

where n is the number of observations. The operation of the plan is described by analyzing the sequence $\{T(n)\}$. Consider the point $T(1) = X_1$ where X_1 is the first sample observation. If T(1) falls on or above an upper bound $T_u(1)$ and low values of X are desirable, the lot is rejected. If T(1) falls on or below a lower bound $T_\ell(1)$, the lot is accepted. If, however, $T_\ell(1) < T(1) < T_u(1)$, another observation is taken. Then $T(2) = X_1 + X_2$ is evaluated in a similar manner, and so on until a final course of action can be determined.

The decision variable T(n) used for testing the mean of a normal distribution with σ'^2 known is derived from Wald's sequential probability ratio test [9]. The test is given as follows. Two points (θ_0, α) and (θ_1, β) are selected where α is the probability of rejecting $\bar{X}'=\theta_0$ and is the probability of accepting $\bar{X}'=\theta_1$. The SPR of the random sample $(X_1, X_2, \ldots, X_n)$ from the normal distribution is

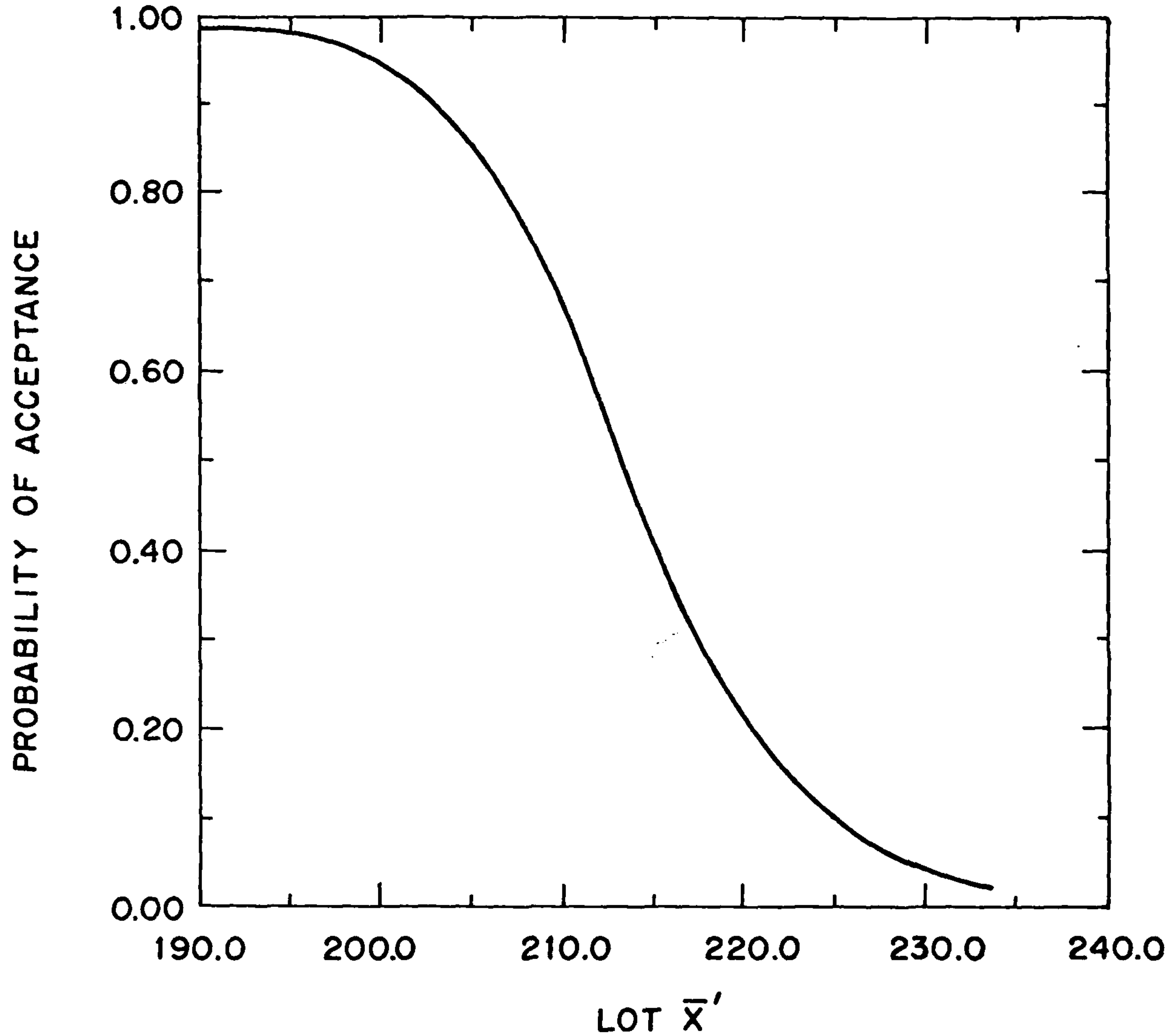

Figure 2. OC Curve: Single Sampling Plan (Sample Problem)

defined as

$$SPR = \frac{\dfrac{1}{(2\pi)^{n/2}\sigma'^{n}} \, e^{-\frac{1}{2\sigma'^2} \sum_{i=1}^{n} (X_i - \theta_1)^2}}{\dfrac{1}{(2\pi)^{n/2}\sigma'^{n}} \, e^{-\frac{1}{2\sigma'^2} \sum_{i=1}^{n} (X_i - \theta_0)^2}} \qquad (4)$$

In theory, the ratio is computed at each stage of the inspection and additional observations are taken as long as

$$B < SPR < A \qquad (5)$$

where B and A are appropriately defined constants. Inspection is terminated with acceptance of the lot if

$$SPR \leq B \qquad (6)$$

Inspection is terminated with rejection of the lot if

$$SPR \geq A \qquad (7)$$

Selecting $A = (1-\beta)/\alpha$, $B = \beta/(1-\alpha)$, taking logarithms of the inequalities and simplifying, (5), (6) and (7) become

$$\frac{\sigma'^2}{(\theta_1-\theta_0)} \log \frac{\beta}{1-\alpha} + n\frac{(\theta_0+\theta_1)}{2} < T(n) = \sum_{i=1}^{n} X_i <$$

$$\frac{\sigma'^2}{(\theta_1-\theta_0)} \log \frac{1-\beta}{\alpha} + n\frac{(\theta_0+\theta_1)}{2} \qquad (8)$$

$$T(n) = \sum_{i=1}^{n} X_i - \frac{\sigma'^2}{\theta_1 - \theta_0} \log \frac{\beta}{1-\alpha} + n \frac{\theta_0 + \theta_1}{2} \tag{9}$$

$$T(n) = \sum_{i=1}^{n} X_i \geq \frac{\sigma'^2}{\theta_1 - \theta_0} \log \frac{1-\beta}{\alpha} + n \frac{\theta_0 + \theta_1}{2} \tag{10}$$

respectively. The decision variable therefore becomes T(n) rather than the SPR.

By using $\bar{X}'_{AQL} = \theta_0$ and $\bar{X}'_{LTPD} = \theta_1$, the limits for the plan from (9) and (10) become

$$T_u(n) = h_1 + S\ n \tag{11}$$

$$T_\ell(n) = h_0 + S\ n \tag{12}$$

where the common slope formed by the limit lines is given by

$$S = \frac{\bar{X}'_{AQL} + \bar{X}'_{LTPD}}{2} \tag{13}$$

and the intercept of the lines when n=0 is given by

$$h_0 = \frac{\sigma'^2 \log B}{\bar{X}'_{LTPD} - \bar{X}'_{AQL}} \tag{14}$$

and

$$h_1 = \frac{\sigma'^2 \log A}{\bar{X}'_{LTPD} - \bar{X}'_{AQL}} \tag{15}$$

The SPR OC and ASN Curves

Wald [9] shows that on an OC curve for $\bar{X}' = -\infty$, $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, $(\bar{X}'_{AQL} + \bar{X}'_{LTPD})/2$, and $+\infty$, the values of $L(X')$ (the P_a of a lot whose mean equals $\bar{X}'$) are as follows:

$$L(-\infty) = 1 \qquad L(\bar{X}'_{AQL}) = 1-\alpha$$

$$L\left(\frac{\bar{X}'_{AQL} + \bar{X}'_{LTPD}}{2}\right) = \frac{\log A}{\log A - \log B} \tag{16}$$

$$L(\bar{X}'_{LTPD}) = \beta \qquad L(-\infty) = \emptyset$$

An approximate general equation is given by

$$L(\bar{X}') \sim \frac{A^h - 1}{A^h - B^h} \tag{17}$$

where

$$h = \frac{\bar{X}'_{AQL} + \bar{X}'_{LTPD} - 2\bar{X}'}{\bar{X}'_{LTPD} - \bar{X}'_{AQL}} \tag{18}$$

Wald also derived an approximate formula for the ASN curve which is given as

$$E_{\bar{X}'}(n) = \frac{h_1 + L(\bar{X}')\,(h_0 - h_1)}{\bar{X}' - S} \tag{19}$$

Sample Illustration

Using the values defined in Chapter I, the number of observations and acceptance and rejection limits are tabulated in Table 1 along with a sample set of observations. The same test plan is graphically illustrated in Figure 3. The corresponding OC and ASN curves are shown by Figure 4 and Figure 5 respectively. As indicated, the plan would terminate with acceptance of the lot at n = 8.

Truncation of the SPR Plan

As mentioned previously, the main disadvantage of the SPR plan is that there is no effective upper limit to the number of items that are required for inspection before a decision to accept or reject a lot can be made. The sample size of an SPR plan is a random variable whose value will occasionally be quite large. Truncation of the procedure is used to force a decision prior to or at $n = n'$ where n' is an a-priori fixed integer. The following is a common rule for accepting or rejecting a particular lot if a decision has not been reached for $n \leq n'$ with the regular sequential procedure. If $T(n') \geq (T_\ell(n') + T_u(n'))/2$ the lot is rejected, and if $T(n') < (T_\ell(n')+T_u(n'))/2$ the lot is accepted.

Several rules of thumb have been suggested for establishing an effective n'. Guild [5] uses n' equal to twice the maximum ASN at $\bar{X}'_{AQL}$ or $\bar{X}'_{LTPD}$. Grant and Leavenworth [4] set n' equal to three times N; the number of samples required by a single sampling plan with the

Table 1. Acceptance and Rejection Limits of Sample Problem SPR Plan With Sample Observations

n	X(n)	T(n)	$T_{\ell}(n)$	$T_{u}(n)$	Decision
1	201	201	131.45	316.55	SAMPLE
2	206	407	343.95	529.05	SAMPLE
3	199	606	556.45	741.55	SAMPLE
4	204	810	768.95	954.05	SAMPLE
5	203	1013	981.45	1166.55	SAMPLE
6	197	1210	1193.95	1379.05	SAMPLE
7	199	1409	1406.45	1591.55	SAMPLE
8	203	1612	1618.95	1804.05	ACCEPT
9			1831.45	2016.55	
10			2043.95	2229.05	
11			2256.45	2441.55	
12			2468.95	2654.05	
13			2681.45	2866.55	
14			2893.95	3079.05	
15			3106.45	3291.55	
16			3318.95	3504.05	
17			3531.45	3716.55	
18			3743.95	3929.05	
19			3956.45	4141.55	
20			4168.95	4354.05	
⋮			⋮	⋮	

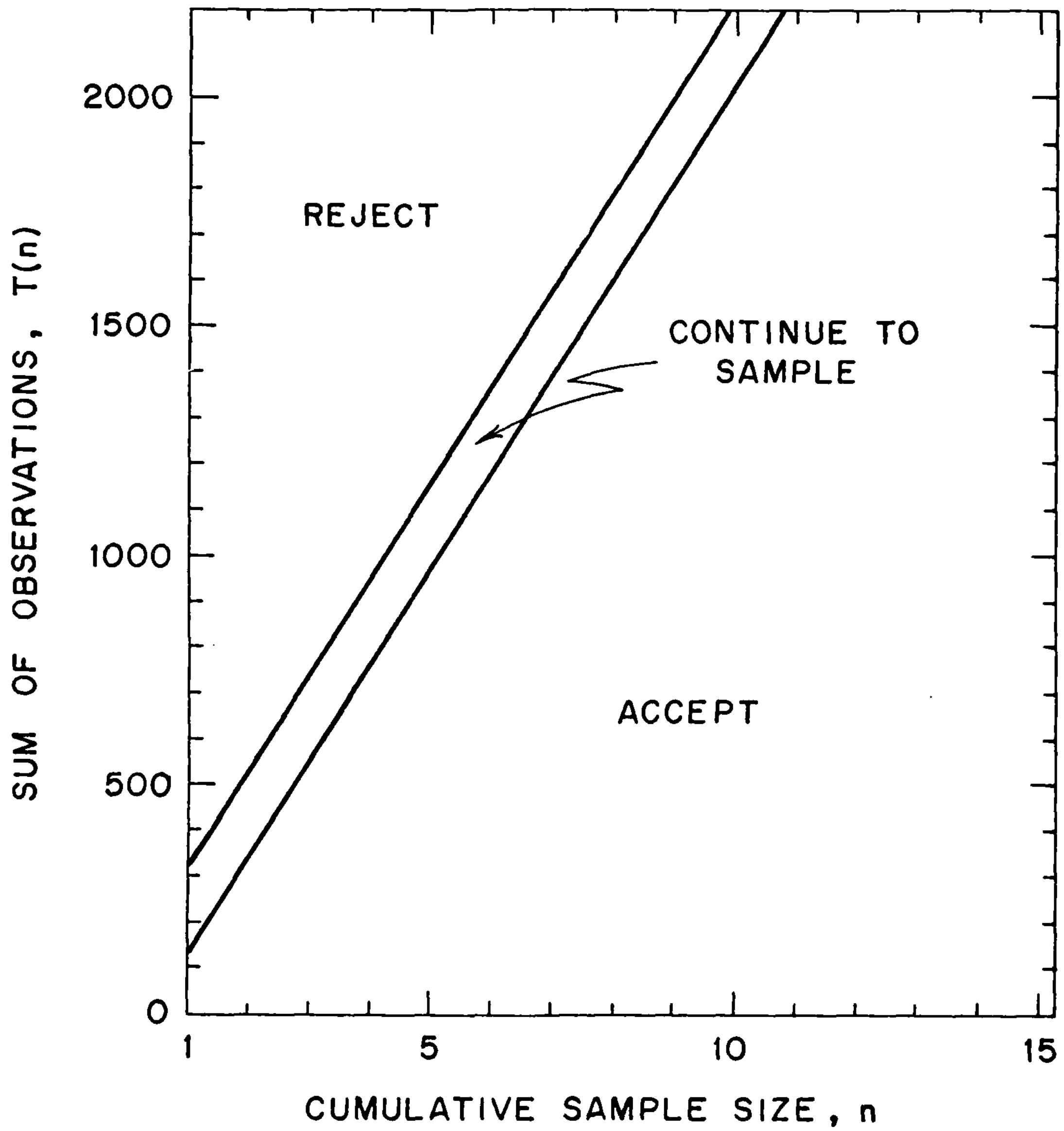

Figure 3. Plot of Sample Problem SPR Plan with Observations

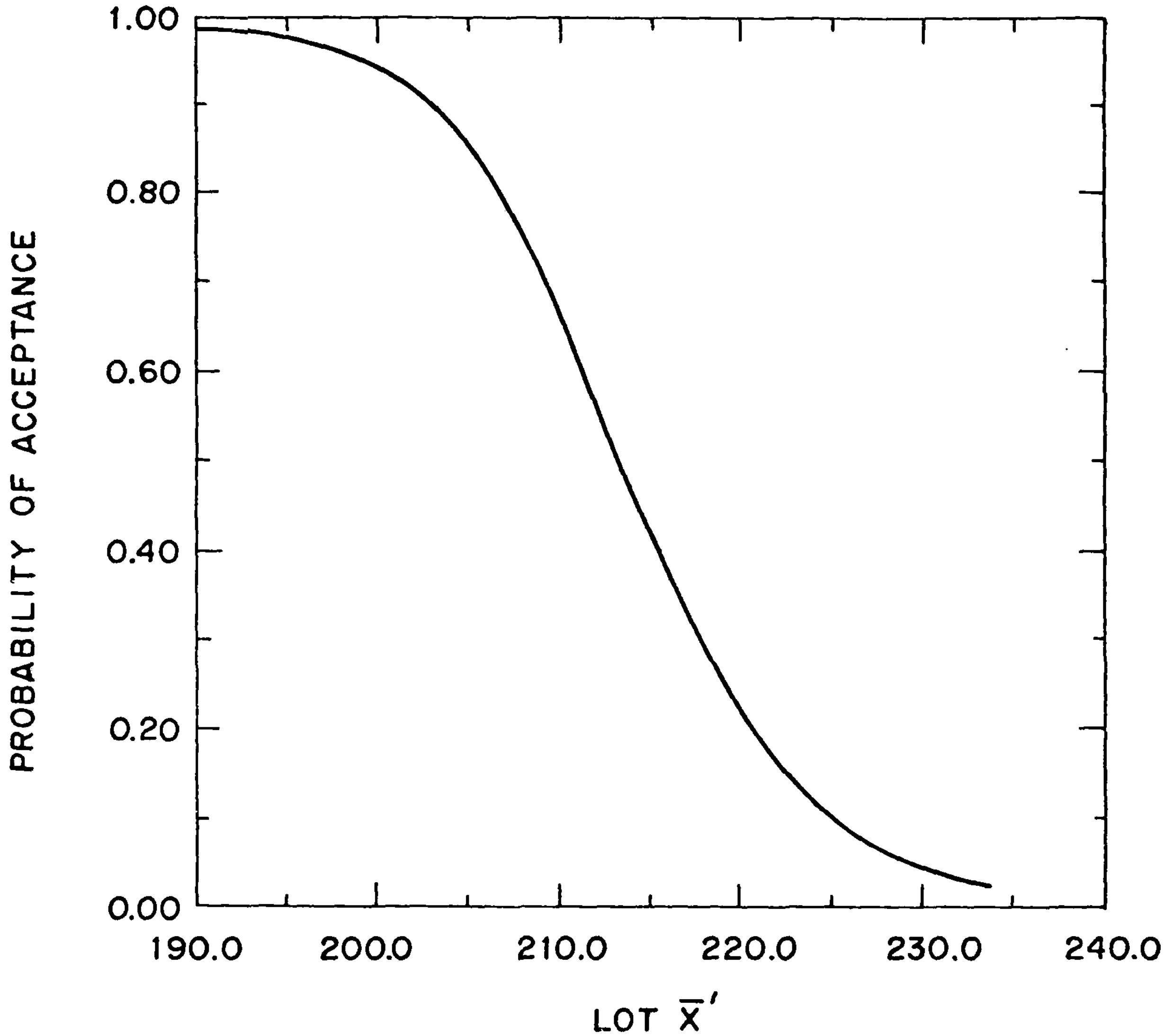

Figure 4. OC Curve: SPR Plan (Sample Problem)

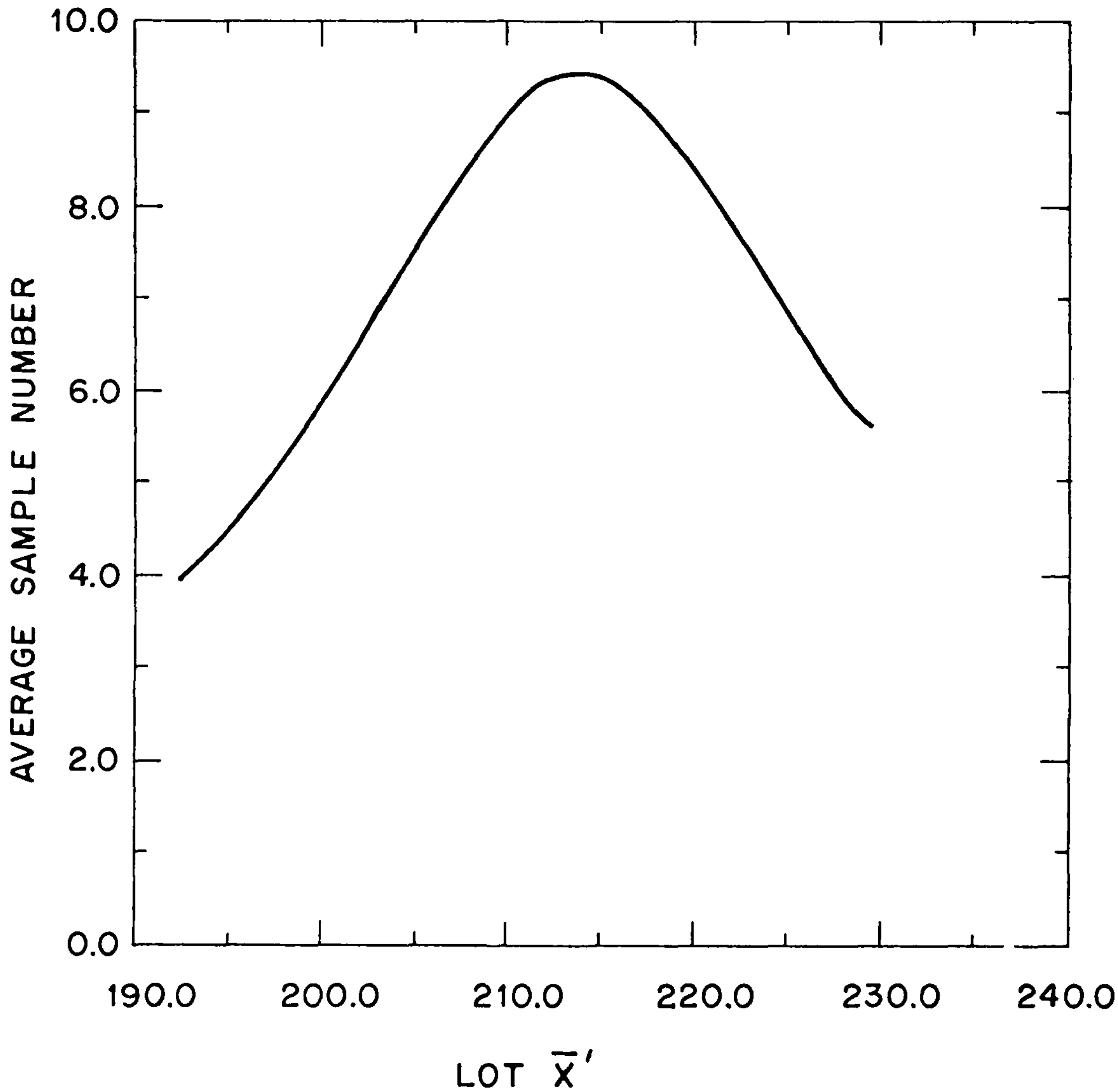

Figure 5. ASN Curve: SPR Plan (Sample Problem)

same level of protection. Wald [9] indicates that with n' put as high as three times the maximum expected value as indicated by the ASN curve, the effect of truncation on the OC curve is negligibly small since the probability is nearly one that the regular sequential procedure will terminate for $n < n'$. For the Wald method of truncation, the value of n' to be used in the sample problem is calculated to be 30. Compared to 13, the number of samples required by the single sampling plan to yield the same level of protection, 30 seems relatively large and possibly prohibitively expensive if the cost of inspection is high.

Table 2 is extracted from Wald's "Sequential Analysis" [9] and illustrates the effect of truncation on the plan for different values of α and β. If the plan is based on the values of α and β shown, but a decision is made at n' even when the regular sequential plan requires a continuation of the process, the realized values $\alpha(n')$ and $\beta(n')$ will not exceed the tabular entries. The table relates to the test of the mean of a normally distributed variate, the difference between the null and alternate hypothesis adjusted for each pair (α, β) so that the number of trials required by the single sampling plan of strength (α, β) is 1000. Notice that as α and β are relaxed, the values of $\alpha(n')$ and $\beta(n')$, even when n' is increased to twice N, vary by 15% when $\alpha = .05$ and $\beta = .05$. Wald's opinion is that the upper limits given in the table are considerably above the true values of $\alpha(n')$ and $\beta(n')$ when n' is not much higher than N. However, any difference between the pairs (α, β) and $(\alpha(n'), \beta(n'))$ may be intolerable. A more reasonable

Table 2. Effect on Risks of Error of Truncating a Sequential Analysis at a Predetermined Number of Trials

Number of Trials	$\alpha = .01$ and $\beta = .01$		$\alpha = .01$ and $\beta = .05$		$\alpha = .05$ and $\beta = .05$	
	Upper bound of effective $\alpha(n')$	Upper bound of effective $\beta(n')$	Upper bound of effective $\alpha(n')$	Upper bound of effective $\beta(n')$	Upper bound of effective $\alpha(n')$	Upper bound of effective $\beta(n')$
1000	.020	.020	.033	.070	.095	.095
1200	.015	.015	.024	.063	.082	.082
1400	.013	.013	.019	.058	.072	.072
1600	.012	.011	.016	.055	.066	.066
1800	.011	.010	.014	.053	.062	.062
2000	.010	.010	.012	.052	.058	.058
2200	.010	.010	.012	.051	.056	.056
2400	.010	.010	.011	.051	.055	.055
2600	.010	.010	.011	.051	.053	.053
2800	.010	.010	.010	.050	.053	.053
3000	.010	.010	.010	.050	.052	.052

plan would be one which guarantees the same level of protection with a commensurate savings in required sample size.

CHAPTER III

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

The SNPR plan is an acceptance sampling procedure bounded by two lines which intersect at $n = n'$, where n' is an a-priori fixed integer. Although the procedures for SPR and SNPR plans are similar, the SNPR plan requires no truncation rules. As mentioned in the previous chapter, truncation of the SPR plan makes the OC curve of the plan less discriminating. Therefore, the implicit property of the SNPR plan may be desirable if it can provide the same level of protection as the non-truncated SPR plan.

The General Approach

For the SNPR plan, $T_\ell(n)$ and $T_u(n)$ are constructed which converge at some maximum number of units, n'. The decision variable for the plan is the same as that defined in equation (1) for the SPR plan. Similarly, the operation of the plan is the same. If no decision is made prior to sampling the n'th unit, the lot is rejected if $T(n') > n'\bar{X}'$ and accepted if $T(n') \leq n'\bar{X}'$. The plan is graphically illustrated in Figure 6.

$T_u(1)$ and $T_\ell(1)$ are determined from

$$T_u(1) = \bar{X}' + Z_{\alpha'}\sigma' \tag{20}$$

$$T_\ell(1) = \bar{X}' - Z_{\alpha'}\sigma' \tag{21}$$

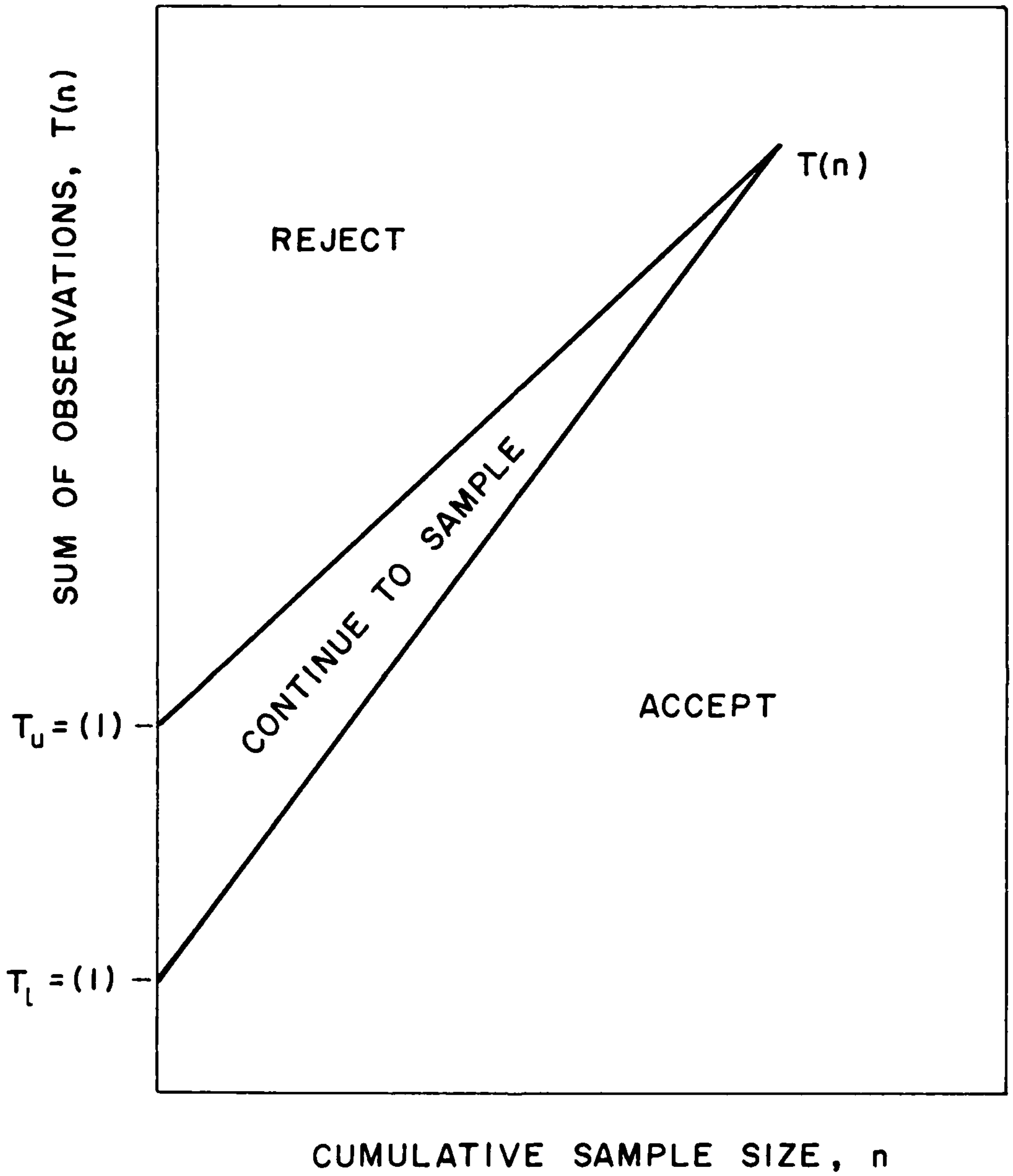

Figure 6. Plot of an SNPR Plan

so that the probability of acceptance (P_a) and the probability of rejection (P_r) are both equal to α' for X' at n = 1. At n = n'

$$T_u(n') = T_\ell(n') = n'\bar{X}' \tag{22}$$

The convergence by the limits on $n'\bar{X}'$ indicates a step by step reduction, d, in the size of the interval $Z_{\alpha'}\sigma'$. This is shown by

$$d = Z_{\alpha'}\sigma'/(n'-1),\ n' > 0 \tag{23}$$

Therefore, the calculation for the upper limit becomes

$$T_u(n) = n\bar{X}' + Z_{\alpha'}\sigma' - (n-1)\ Z_{\alpha'}\sigma'/(n'-1)$$

or (24)

$$T_u(n) = n\bar{X}' + (n'-n)Z_{\alpha'}\sigma'/(n'-1),\ 1 < n < n'$$

Similarly,

$$T_\ell(n) = n\bar{X}' - (n'-n)Z_{\alpha'}\sigma'/(n'-1),\ 1 < n < n' \tag{25}$$

$\bar{X}'$ and $Z_{\alpha'}$ for the SNPR plan must now be established so that limits which provide the same level of protection as the SPR plan may be calculated.

Sample Illustration

Using the example, if $T_u(1)$ and $T_\ell(1)$ are arbitrarily set to the same values as in the SPR plan, $\bar{X}'$ and $Z_{\alpha'}$ from equations (20) and (21) become

$$\bar{X}' = \frac{T_u(1) + T_\ell(1)}{2} = \frac{316.55 + 131.46}{2} = 224.01$$

$$Z_{\alpha'} = \frac{316.55 - 224.01}{2} = 3.0840, \ \alpha' = .0010$$

Table 3 shows the proposed plan for n' = 10 (10 is the maximum ASN of the SPR plan) while Figure 7 shows the plot.

α and β for the SNPR plan are calculated in the following manner. Since $T(n) = \sum_{i=1}^{n} X_i$ and X is $N(\bar{X}', \sigma'^2)$, T(n) is $N(n\bar{X}', n\sigma'^2)$ and $\sigma(T(n)) = \sqrt{n}\sigma'$. Therefore, the normal deviate

$$Z = (T_u(n) - n\bar{X}')/\sqrt{n}\sigma'$$

can be used to solve for α. Table 4 lists the results of calculations. Using the equation

$$\alpha = \alpha_1 + (1-\alpha_1-\beta_1)\alpha_2 + (1-\alpha_1-\beta_1)(1-\alpha_2-\beta_2)\alpha_3 + \ldots + \ldots (1-\alpha_9-\alpha_9)\alpha_{10}$$

where $\alpha_k = P(Z>Z_{\alpha_n} | \bar{X}'=\bar{X}'_{AQL})$ and $\beta_k = P(Z<Z_{\beta_n} | \bar{X}'=\bar{X}'_{LTPD})$, α is calculated to be .005. Similarly β is calculated to be .450. Figure 8 shows the OC curves for the corresponding SPR and SNPR plans.

The arbitrary values of $\bar{X}'$ and $Z_{\alpha'}$ chosen for the SNPR plan yield an OC curve which is obviously less discriminating than that of the SPR plan. However, altering the values of $\bar{X}'$, $Z_{\alpha'}$ and n' changes the values of α and β. It will be shown that properly chosen values of $\bar{X}'$, $Z_{\alpha'}$ and n' will yield an OC curve for the SNPR plan which discriminates as well as the OC curve of the SPR plan.

Table 3. Acceptance and Rejection Limits of Sample Problem SNPR Plan ($\bar{X}'$ = 224.01, $Z_{\alpha'}$ = 3.0840 and n' = 10) With Sample Observations

n	X(n)	T(n)	$T_\ell(n)$	$T_u(n)$	Decision
1	201	201	131.49	316.53	SAMPLE
2	206	407	365.78	530.26	SAMPLE
3	199	606	600.07	743.99	SAMPLE
4	204	810	834.36	957.72	ACCEPT
5			1068.65	1171.45	
6			1302.94	1385.18	
7			1537.23	1598.91	
8			1771.52	1812.64	
9			2005.81	2026.37	
10			2240.10	2240.10	

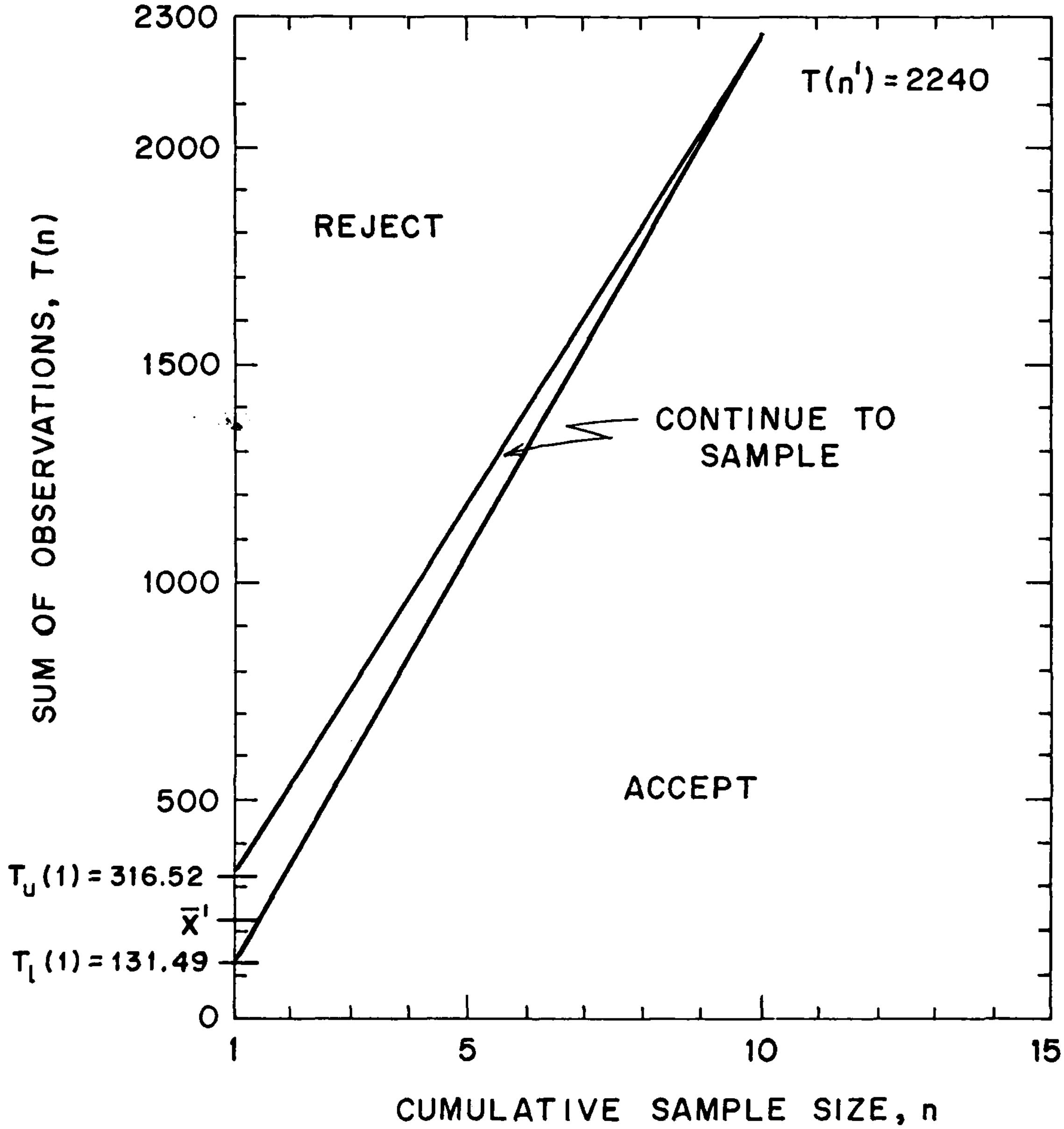

Figure 7. Plot of Sample Problem SNPR Plan ($\bar{X}'$ = 224.01, $Z_{\alpha'}$ = 3.0840 and n' = 10)

Table 4. Calculations Used to Solve for α and β in Sample Problem SNPR Plan ($\bar{X}' = 224.01$, $Z_{\alpha'} = 3.0840$ and $n' = 10$

				Calculations for α						Calculations for β					
n	$\sqrt{n}\sigma'$	$T_\ell(n)$	$T_u(n)$	$n\bar{X}'$	Z_{α_n}	α_n	Z_{β_n}	β_n	$1-\alpha_n-\beta_n$	$n\bar{X}'$	Z_{α_n}	α_n	Z_{β_n}	β_n	$1-\alpha_n-\beta_n$
1	30	131.49	316.53	200	3.88	.0000+	-2.28	.0112	.9887	225	3.05	.0011	-3.12	.0009	.9979
2	42.43	365.78	530.25	400	3.07	.0010	-.80	.2099	.7801	450	1.89	.0292	-1.98	.0236	.9979
3	51.96	600.07	743.98	600	2.77	.0027	.00+	.5005	.3874	675	1.33	.0921	-1.44	.0746	.7876
4	60.00	834.36	957.72	800	2.63	.0042	.57	.7165	.1081	900	.96	.1680	-1.09	.1370	.5474
5	67.08	1068.65	1171.45	1000	2.55	.0052	1.02	.8469	.0160	1125	.69	.2443	-.84	.2004	.3039
6	73.48	1302.94	1385.18	1200	2.52	.0058	1.40	.9194	.0012	1350	.49	.3160	-.64	.2609	.1285
7	79.37	1537.23	1598.91	1400	2.50	.0061	1.72	.9580	0.0	1575	.30	.3816	-.47	.3171	.0387
8	84.85	1771.52	1812.64	1600	2.50	.0061	2.02	.9783	0.0	1800	.15	.4408	-.33	.3686	.0074
9	90.00	2005.81	2026.37	1800	2.51	.0059	2.29	.9889	0.0	2025	.01	.4939	-.21	.4156	.0006
10	94.86	2240.10	2240.10	2000	2.53	.0056	2.53	.9943	0.0	2250	-.10	.5415	-.10	.4584	0.0

$$\alpha = \alpha_1 + (1-\alpha_1-\beta_1)\,\alpha_2 + (1-\alpha_1-\beta_1)(1-\alpha_2-\beta_2)\,\alpha_3 + \dots + \dots (1-\alpha_9-\beta_9)\,\alpha_{10}$$
$$= .0000 + (.9887)(.0010) + (.9887)(.7801)(.0027) + \dots + \dots (0)\ .0056$$
$$= .0056$$

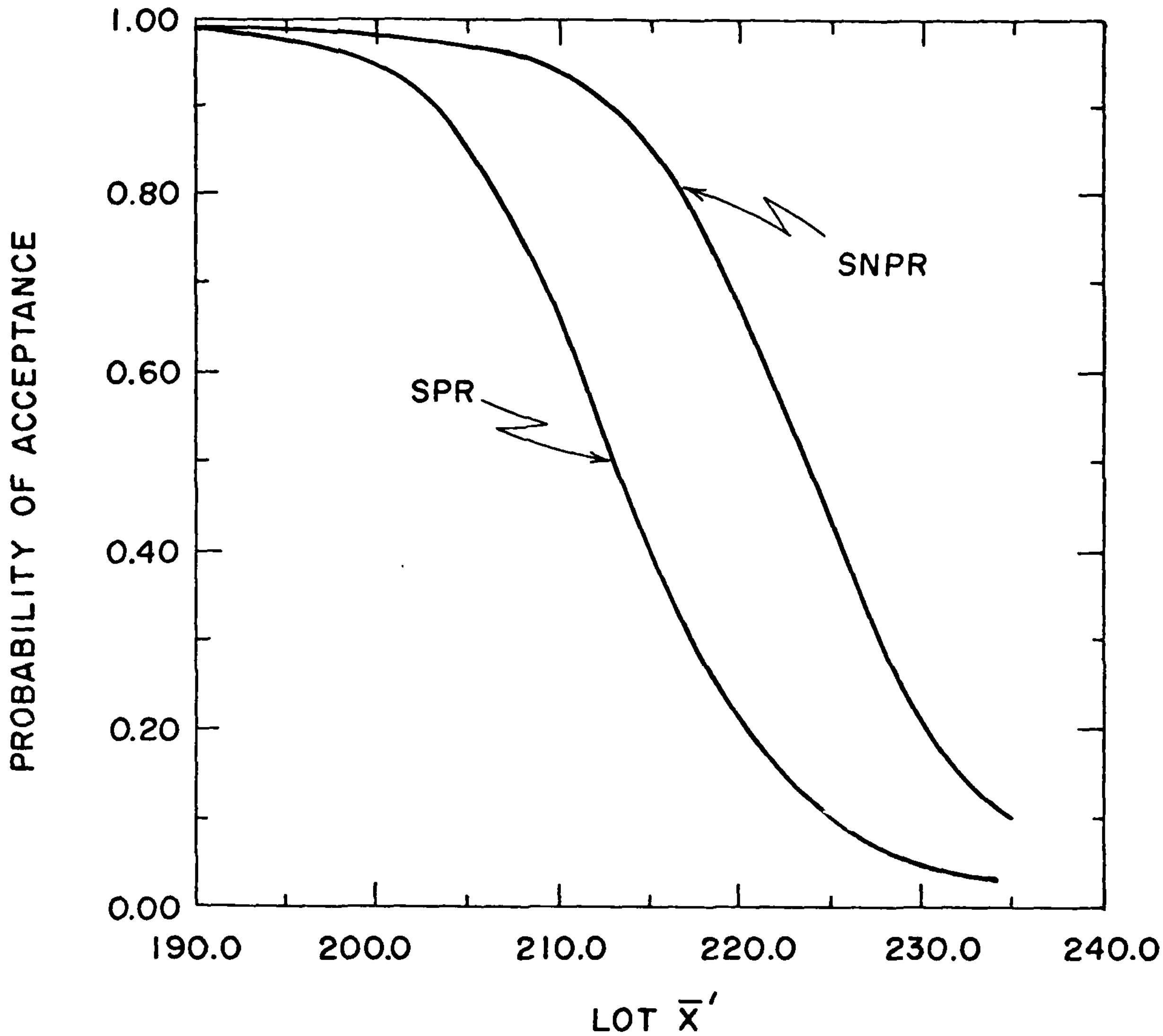

Figure 8. OC Curves: SNPR Plan (Sample Problem; $\bar{X}'$ = 224.01, $Z_{\alpha'}$ = 3.0840 and n' = 10) and SPR Plan (Sample Problem)

Preliminary Analysis

Holding n' and $Z_{\alpha'}$ constant, $\bar{X}'$ is altered to examine the effect on α and β. Table 5 lists the results. It is observed that as $\bar{X}'$ decreases, α increases and β decreases. This is intuitively reasonable because altering $\bar{X}'$ relocates the envelope formed by the acceptance and rejection limits. A downward relocation of the envelope, caused by decreasing $\bar{X}'$, increases the rejection region. If the rejection region is increased, the probability of rejecting acceptable material is increased, therefore, α is increased. Similarly β decreases as $\bar{X}'$ decreases. This is graphically illustrated in Figure 9. OC curves for the same plans are compared to the SPR plan's OC curve in Figure 10. A strong similarity exists between the OC curves of the SPR plan and the SNPR plan with $\bar{X}' = 214.05$. This particular value of $\bar{X}'$has not been chosen arbitrarily. It is the value of $\bar{X}_a$, the acceptance limit for the single sampling plan discussed in Chapter II. $\bar{X}_a$ will be used in the development of a heuristic procedure which establishes $\bar{X}'$ for an SNPR plan equivalent to the SPR plan.

Varying $Z_{\alpha'}$ and holding $\bar{X}'$ and n' constant effects the values of α and β as shown in Table 6. As $Z_{\alpha'}$ increases, α and β both decrease. Again this is intuitively reasonable because increasing $Z_{\alpha'}$ widens the envelope formed by the acceptance and rejection limits. The widening decreases both the acceptance and rejection regions, as shown in Figure 11, thereby decreasing both α and β. Figure 12 shows the corresponding OC curves. Again a similarity exists between the OC

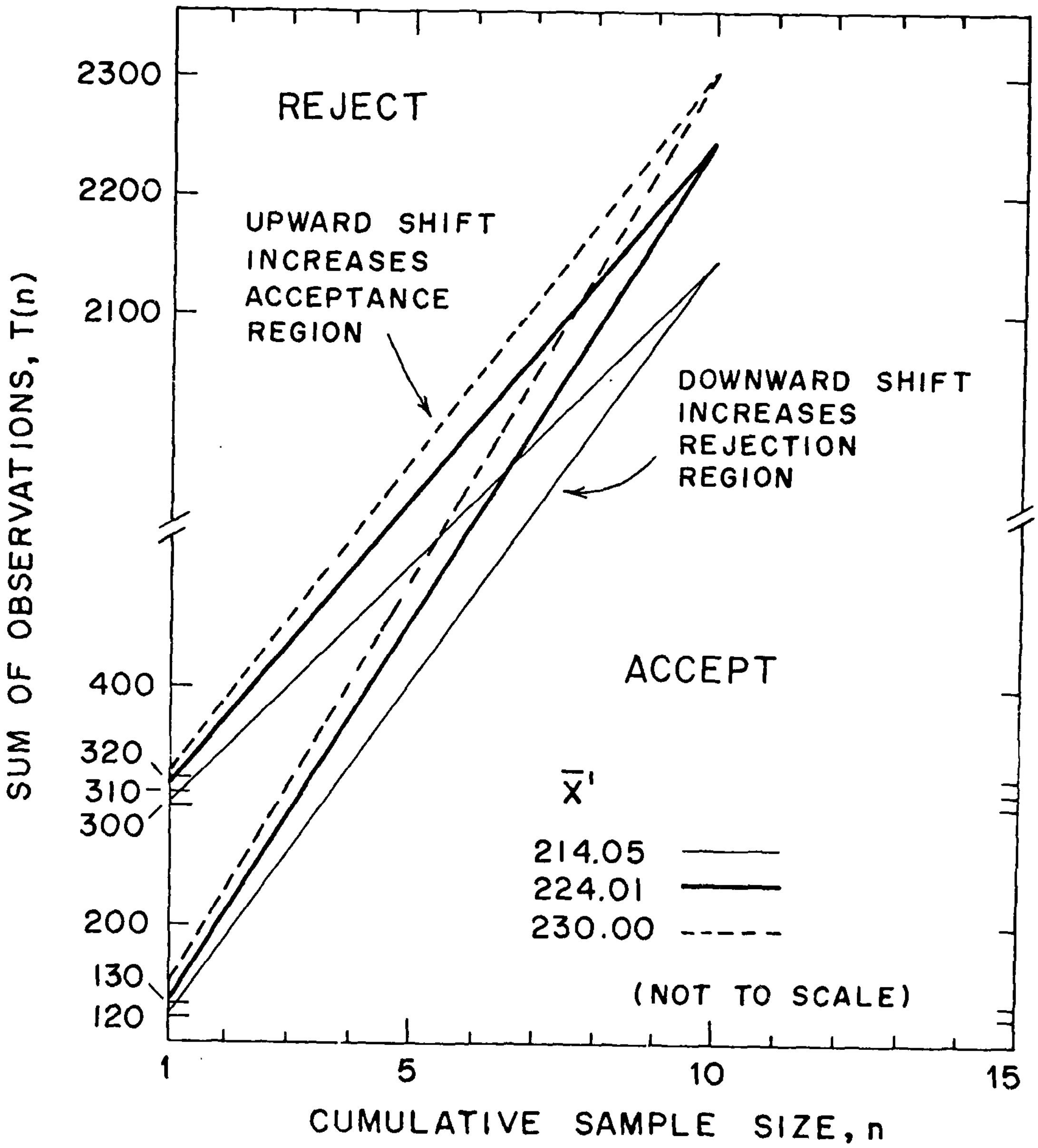

Figure 9. Plot of Sample Problem SNPR Plans ($Z_{\alpha'}$ = 3.0840, n' = 10 and $\bar{X}'$ = 214.05, 224.01 and 230.00)

Table 5. Effect on α and β of Altering $\bar{X}'$ (Sample Problem SNPR Plan: $Z_{\alpha'}$ = 3.0840, n' = 10)

$\bar{X}'$	$T_\ell(1)$	$T_u(1)$	T(n')	α	β
214.05	121.53	306.57	2140.50	.001	.734
224.01	131.49	316.53	2240.09	.005	.450
230.00	137.48	322.52	2300.00	.050	.094

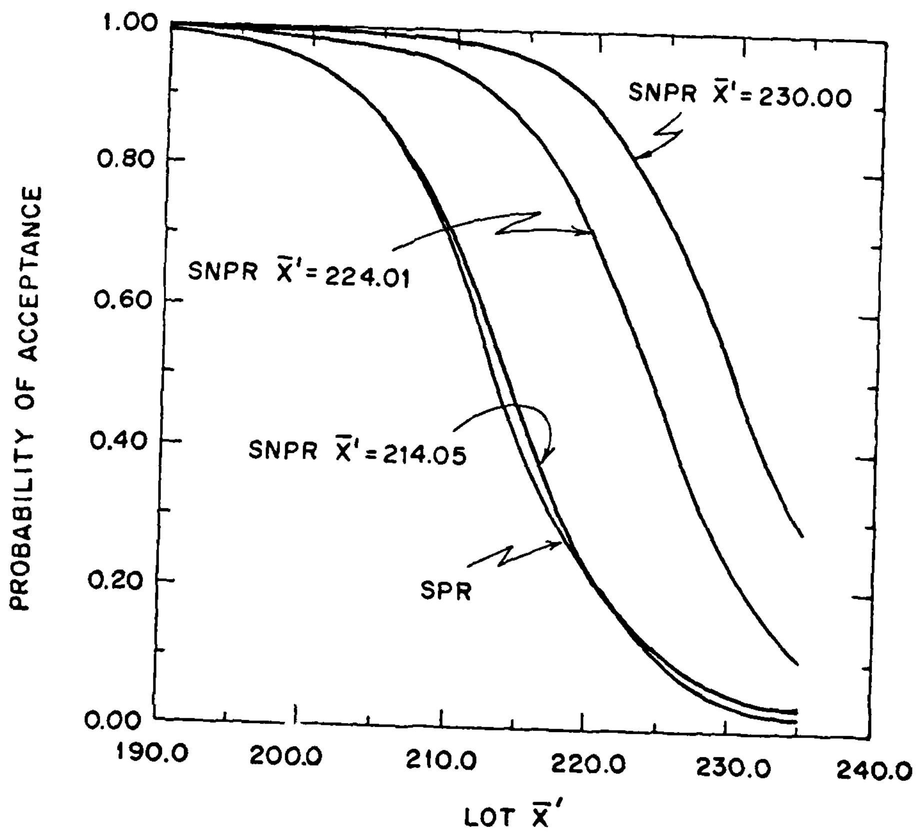

Figure 10. OC Curves: SNPR Plan (Sample Problem; $Z_{\alpha'}$ = 3.0840, n' = 10 and $\bar{X}'$ = 214.05, 224.01 and 230.00) and SPR Plan (Sample Problem)

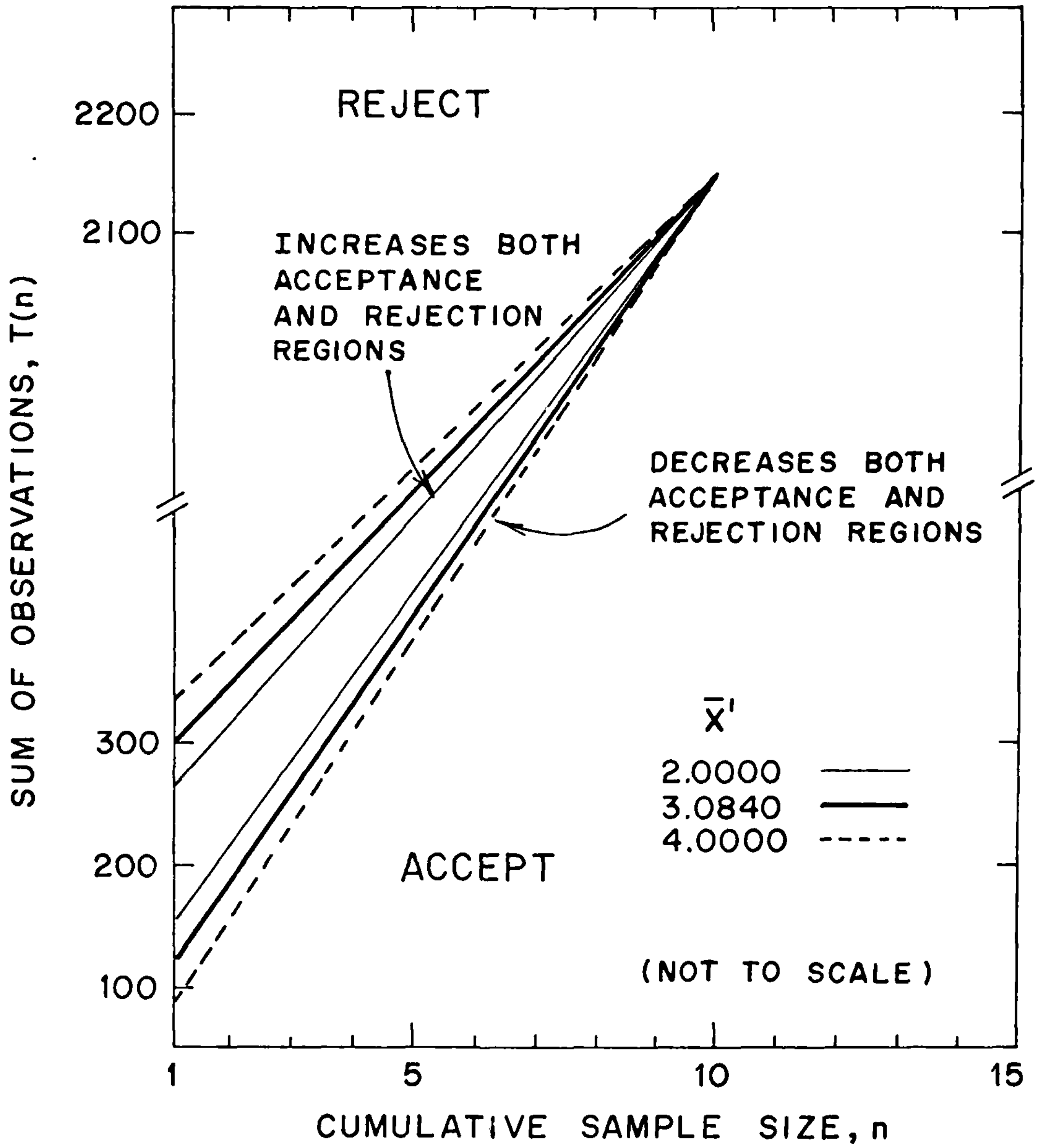

Figure 11. Plot of Sample Problem SNPR Plans ($\bar{X}'$ = 214.05, n' = 10 and $Z_{\alpha'}$ = 2.0000, 3.0840 and 4.0000)

Table 6. Effect on α and β of Altering $Z_{\alpha'}$ (Sample Problem SNPR Plan: $\bar{X}' = 214.05$, $n' = 10$)

$Z_{\alpha'}$	$T_\ell(1)$	$T_\mu(1)$	$T(n')$	α	β
2.000	154.05	274.04	2140.50	.086	.137
3.084	121.53	306.57	2140.50	.050	.094
4.000	94.05	334.04	2140.50	.036	.075

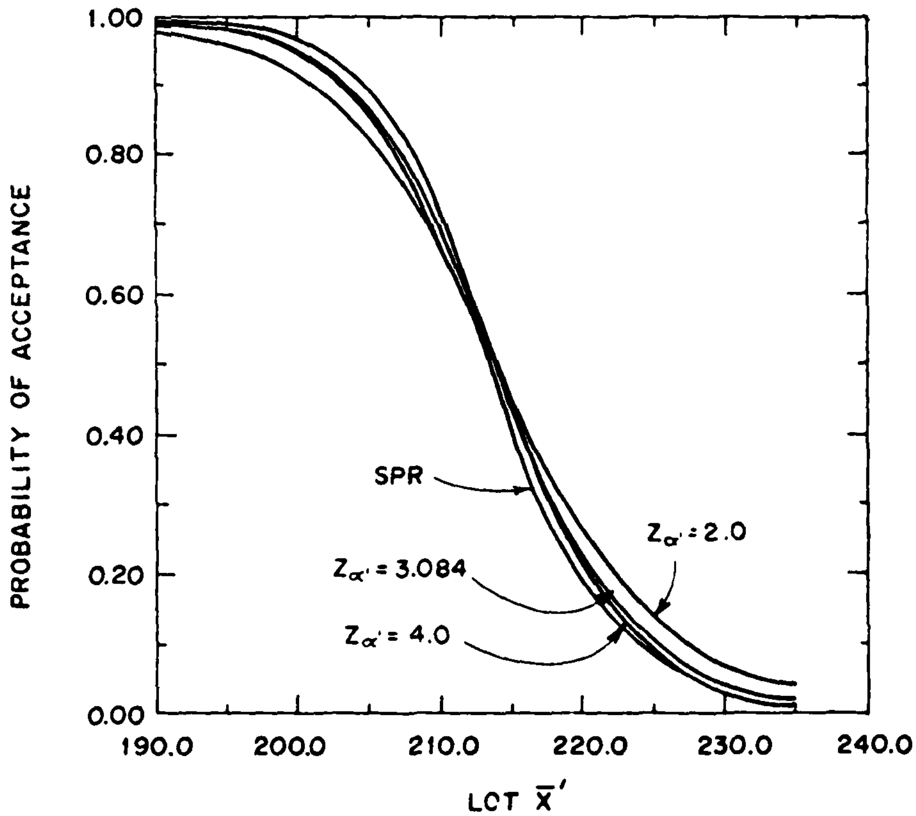

Figure 12. OC Curves: SNPR Plans (Sample Problem; $\bar{X}' = 214.05$, $n' = 10$ and $Z_{\alpha'} = 2.0000$, 3.0840 and 4.0000) and SPR Plan

curves of the SPR plan and the SNPR plan with $\bar{X}'$= 214.05 and $Z_{\alpha'}$ = 3.0840. These values are used in the development of a heuristic which for a specified value of n' establishes $\bar{X}'$ and $Z_{\alpha'}$ for an SNPR plan which provides the same level of protection as the SPR plan.

Development of the Heuristic

Three important facts mentioned in the previous section are the cornerstones of the heuristic procedure:

(1) The OC curve of an SNPR plan with $\bar{X}'$ equal to $\bar{X}_a$ of the single sampling plan, $Z_{\alpha'}$ equal to the $Z_{\alpha'}$ obtained from the limits at n=1 of the SPR plan and n' equal to the maximum ASN of the SPR plan closely approximates the OC curve of the SPR plan.

(2) Increasing the value of $\bar{X}'$ in an SNPR plan decreases the value of α and increases β. Decreasing $\bar{X}'$ has the opposite effect.

(3) Increasing the value of $Z_{\alpha'}$ in an SNPR plan decreases both α and β. Decreasing $Z_{\alpha'}$ has the opposite effect.

Using this information, $Z_{\alpha'}$ and $\bar{X}'$ are altered while holding n' constant at 10. By setting $\bar{X}'$ = 214.24 and $Z_{\alpha'}$ = 3.0100, the calculated values of α and β are .05 and .10 respectively. If n is reduced to 9 (a 10% sampling reduction), the values of $\bar{X}'$ and $Z_{\alpha'}$ need to yield α = .05 and β = .10 are 214.22 and 3.2400 respectively. Table 7 lists the necessary values of $\bar{X}'$ and $Z_{\alpha'}$ for different values of n'. Figure 13 shows the plots of two plans along with a plot of the SPR plan. Notice that as n' decreases, the plan compensates by being less discriminating in its earlier stages. Computational experience indicates that SNPR

Table 7. Values of $\bar{X}'$ and $Z_{\alpha'}$ Necessary to Obtain $\alpha = .05$ and $\beta = .10$ for Varying Values of n' in the Sample Problem

n'	$\bar{X}'$	$Z_{\alpha'}$
10	214.24	3.01
9	214.22	3.24
8	214.19	3.65
7	214.14	4.71

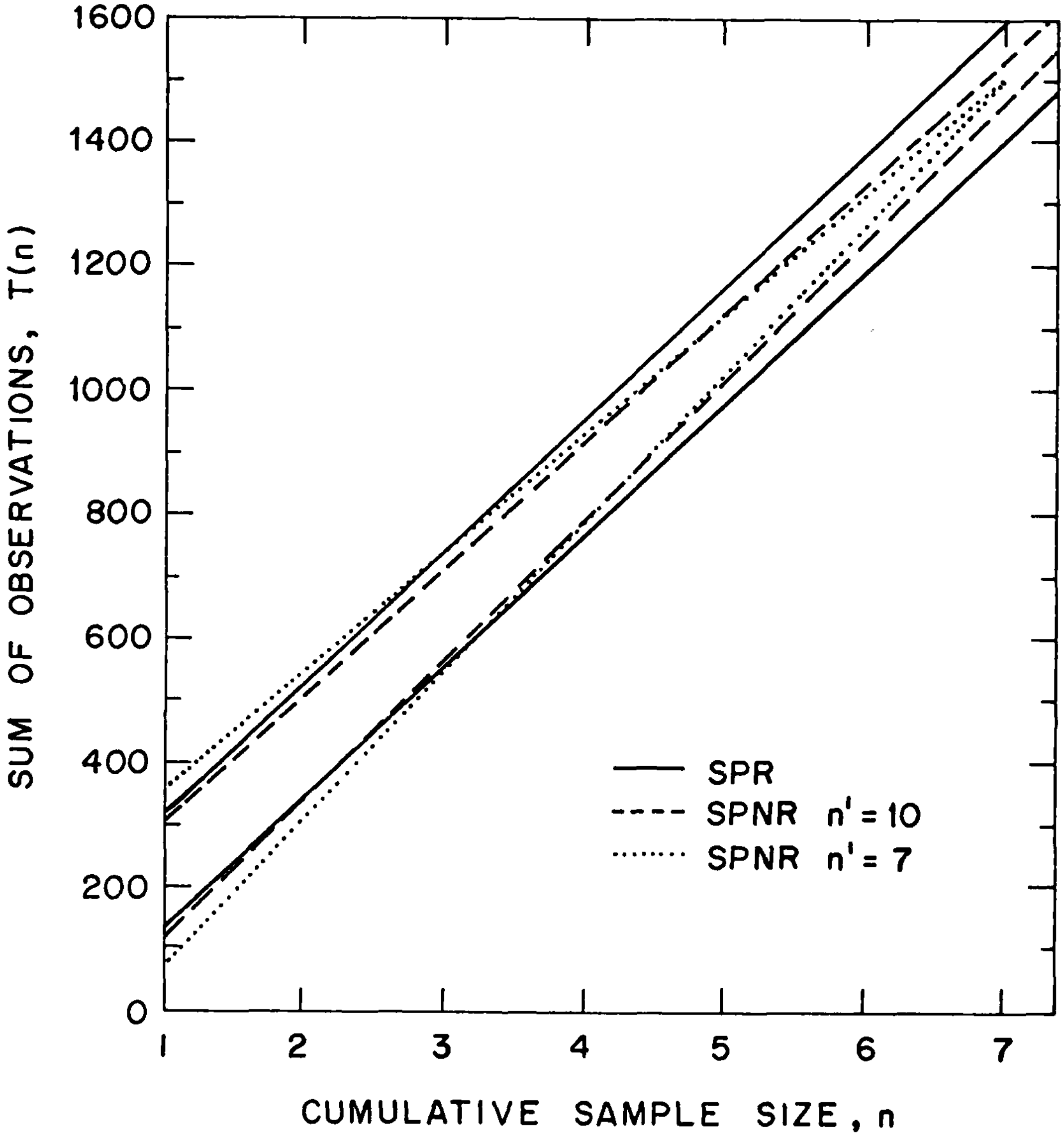

Figure 13. Plots of Sample Problem SNPR Plans (n' = 7 and n' = 10) and Sample Problem SPR Plan

plans with n' less than 70% of the SPR plan's maximum ASN cannot yield an OC curve which approximates that of the SPR plan, regardless of the values of $\bar{X}'$ and $Z_{\alpha'}$ chosen.

Using the estimates of $\bar{X}'$ and $Z_{\alpha'}$ described at the beginning of this section, a more exacting OC curve can be obtained using the following steps:

(1) If n' is chosen other than the maximum ASN of the SPR plan (Nmax), a modified estimate of $Z_{\alpha'}$ is used. This value is obtained by increasing (decreasing) the initial estimate by the percentage decrease (increase) in Nmax:

$$Z_{\alpha'}(n') = Z_{\alpha'}(Nmax)(1+(1-n'/Nmax)) \qquad (26)$$

(2) Examine the values of α and β obtained by using the estimated parameters in the SNPR plan. Determine if they must be increased or decreased to match the objective values of α and β from the SPR plan.

(3) If both α and β must be increased (decreased) to meet the objective values of α and β from the SPR plan, decrease (increase) $Z_{\alpha'}$.

(4) If one (α or β) must be increased and the other decreased to meet the objective values of α and β from the SPR plan, alter $\bar{X}'$ accordingly.

(5) Repeat steps 2 through 4 until the values of α and β from the SNPR test are within the desired limits.

Computational experience with the heuristic, as coded in Appendix A, reveals that usually only two or three iterations of the procedure are needed to obtain α and β within a tolerance of .001. Appendix B contains

several SNPR plans which were generated using the coded heuristic. The associated problem parameters were varied through specific ranges to demonstrate the code's flexibility. In each of the twelve plans, the exact objective values of α and β were met.

Using the sample problem, the following is an example of the heuristic procedure for n' = 8.

(1) Initial Estimates

(Step 1) $\bar{X}' = 214.05 \quad Z_{\alpha'} = 3.0849$

$Z_{\alpha'}(8) = 3.0849\ (1+(1-8/10)) = 3.7019$

(2) With $\bar{X}' = 214.05$, $Z_{\alpha'} = 3.7019$ and n' = 8

(Step 2) $\alpha = .0507 \quad \beta = .0965$

(3) Since α is above the objective value of .05 and β is below

(Step 4) the objective value of .10, $\bar{X}'$ is increased. (The rule for altering $\bar{X}'$ is to first alter it by $\sigma'/100$. After recalculating the values of α and β, determine the proportionate adjustment to $\bar{X}'$ which yields α and β closest to the objective values.)*

With $\bar{X}' = 214.35$, $Z_{\alpha'} = 3.7019$ and n' = 8

$\alpha = .0474 \quad \beta = .1023$

a proportionate adjustment yields $\bar{X}' = 214$

With $\bar{X}' = 214$, $Z_{\alpha'} = 3.7019$ and n' = 8

$\alpha = .0495 \quad \beta = .0993$

(4) With α and β within .001 of the objective values of .05 and .10, respectively, the heuristic procedure is terminated.

---

*The rule for altering $Z_{\alpha'}$ is to increase (decrease) the value of $Z_{\alpha'}$ accordingly, recalculate the values of α and β, then determine the proportionate adjustment to $Z_{\alpha'}$ which yields α and β closest to the objective values.

CHAPTER IV

COMPARISON OF SPR AND SNPR PLANS

Because it reveals the discriminating power of a sampling plan, the OC curve is useful in evaluating alternate sampling plans. In the previous chapter a method to closely match the OC curves of the subject plans was presented and therefore voids any comparisons based on that feature. However, situations may exist where it is advantageous to quickly establish a sequential sampling plan which guarantees truncation after relatively few samples have been taken, even if the cost of truncation is a slightly altered OC curve. Two methods are available to establish a "fast truncated" sequential sampling plan. They are the truncated SPR plan described in Chapter II and the SNPR plan which uses the initial estimates of $\bar{X}'$ and Z_{α}' ($\bar{X}_a$ from the single sampling plan and Z_{α}' derived using the initial acceptance and rejection limits of the SPR plan) discussed in Chapter III.

Four sampling plans are presented in Table 8. The plans are the truncated SPR plan (n' = 10), the non-truncated SPR plan, the SNPR plan (n' = 10) generated using the coded heuristic procedure in Appendix A and the SNPR plan (n' = 10) generated using the initial estimates mentioned above. The first plan (truncated SPR) has $\alpha(10) = .119$ and $\beta(10) = .156$ (these values are upper limits for $\alpha(n')$ and $\beta(n')$ computed via equations developed by Wald [9]). While this is a "fast truncated" sequential plan, the resulting distortion of the OC curve leaves the practicality of this plan highly suspect. The fourth plan (SNPR generated using initial estimates) has $\alpha = .050$ and $\beta = .095$.

Table 8. Comparison of Sequential Plans Using Sample Problem

	TRUNCATED SPR $n'=10$ $\alpha(n')=.119$ $\beta(n')=.156$		SPR $\alpha=.050$ $\beta=.100$		SNPR $n'=10$ $\alpha=.050$ $\beta=.100$		SNPR (EST.) $n'=10$ $\alpha=.050$ $\beta=.095$	
n	$T_\ell(n)$	$T_u(n)$	$T_\ell(n)$	$T_u(n)$	$T_\ell(n)$	$T_u(n)$	$T_\ell(n)$	$T_u(n)$
1	131.45	3.6.55	131.45	316.55	123.97	304.54	121.53	306.57
2	343.95	529.05	343.95	529.05	348.26	508.77	345.86	510.34
3	556.45	741.55	556.45	741.55	572.55	712.99	570.19	714.11
4	768.95	954.05	768.05	954.05	796.83	917.21	794.52	917.88
5	981.45	1166.55	981.45	1166.55	1021.12	1121.44	1018.85	1121.65
6	1193.95	1379.05	1193.95	1379.05	1245.41	1325.66	1243.18	1325.42
7	1406.45	1591.55	1406.45	1591.55	1469.70	1529.89	1467.51	1529.19
8	1618.95	1804.05	1618.95	1804.05	1693.98	1734.11	1691.84	1732.96
9	1831.45	2016.55	1831.45	2016.55	1918.27	1938.34	1916.17	1936.73
10	2043.95	2229.05	2043.95	2229.05	2142.56	2142.56	2140.50	2140.50
11			2256.45	2441.55				
12			2468.95	2654.05				
13			2681.45	2866.55				
14			⋮	⋮				
15								

The OC curve of this plan, although not exactly matched to the OC curve of the non-truncated SPR plan, may reveal sufficient protection that it would not be necessary to perform the heuristic procedure to obtain a more exacting OC curve.

If exact discriminating power of a plan is an absolute requirement, Duncan [1] suggests the relative efficiency of plans with matching OC curves may be determined by examining the amount of inspection required by each plan. The ASN curve of the SPR plan shown in Figure 5 is characteristic of all SPR plans. It can be seen that decisions (generally to accept) are quickly made, on the average, when good quality lots are submitted and that decisions (generally to reject) are quickly made, on the average, when bad lots are submitted. Lots of mediocre quality require more inspection, on the average [7]. The phrase, "on the average" is stressed because the required sample size is, as mentioned previously, a random variable capable of taking on values well above the average.

Although no analytic function has been developed which describes or approximates an SNPR plan's ASN curve, it is relatively easy to envision its rudimentary shape. It is known that values on the curve cannot exceed n' but will most likely take on values well below n'. Therefore, n' represents an upper limit for the curve. In addition, Wald has proven that the efficiency of the SPR test is, "if not exactly, very nearly equal to 1 under H_0 as well as under H_1 [9]." This means that the values on the SPR test's ASN curve at $\bar{X}'_{AQL}$ and $\bar{X}'_{LTPD}$ are lower limits for all sequential tests. Next, shown in

Figure 13 are plots of three sampling plans developed from the sample problem. In the initial stages, n = 1 through 4, the SNPR plan (n' = 10) is as discriminating as the SPR plan while the SNPR plan (N' = 7) is less discriminating. This suggests that the ASN curve of the SNPR plan (n' = 10) closely resembles that of the SPR plan at the tails of the ASN curve. Conversely, the ASN curve of the SNPR plan (n' = 7) is above that of the SPR plan at the tails of the ASN curve. If it is assumed that incoming lots of mediocre quality force the SNPR plan's ASN curve to approach n', the ASN curves in Figure 14 are rough approximations of what can reasonably be expected.

Examining the curves in Figure 14, an expected feature of the SNPR plan becomes evident. As n' decreases, so does the maximum average sample number. However, the curve becomes less peaked; the average sample number increases at values of $\bar{X}'$ which indicate lots of very good or very bad quality. At this point, a trade-off must be made when selecting which plan to use. If one is reasonably sure that the values of $\bar{X}'$ are clustered near one value, the selection of a plan can be made by comparing ASN curves and selecting the plan with the lowest ASN at that point. If the quality of incoming lots is unknown, the cost of inspection versus the number of inspections trade-off analysis must be made.

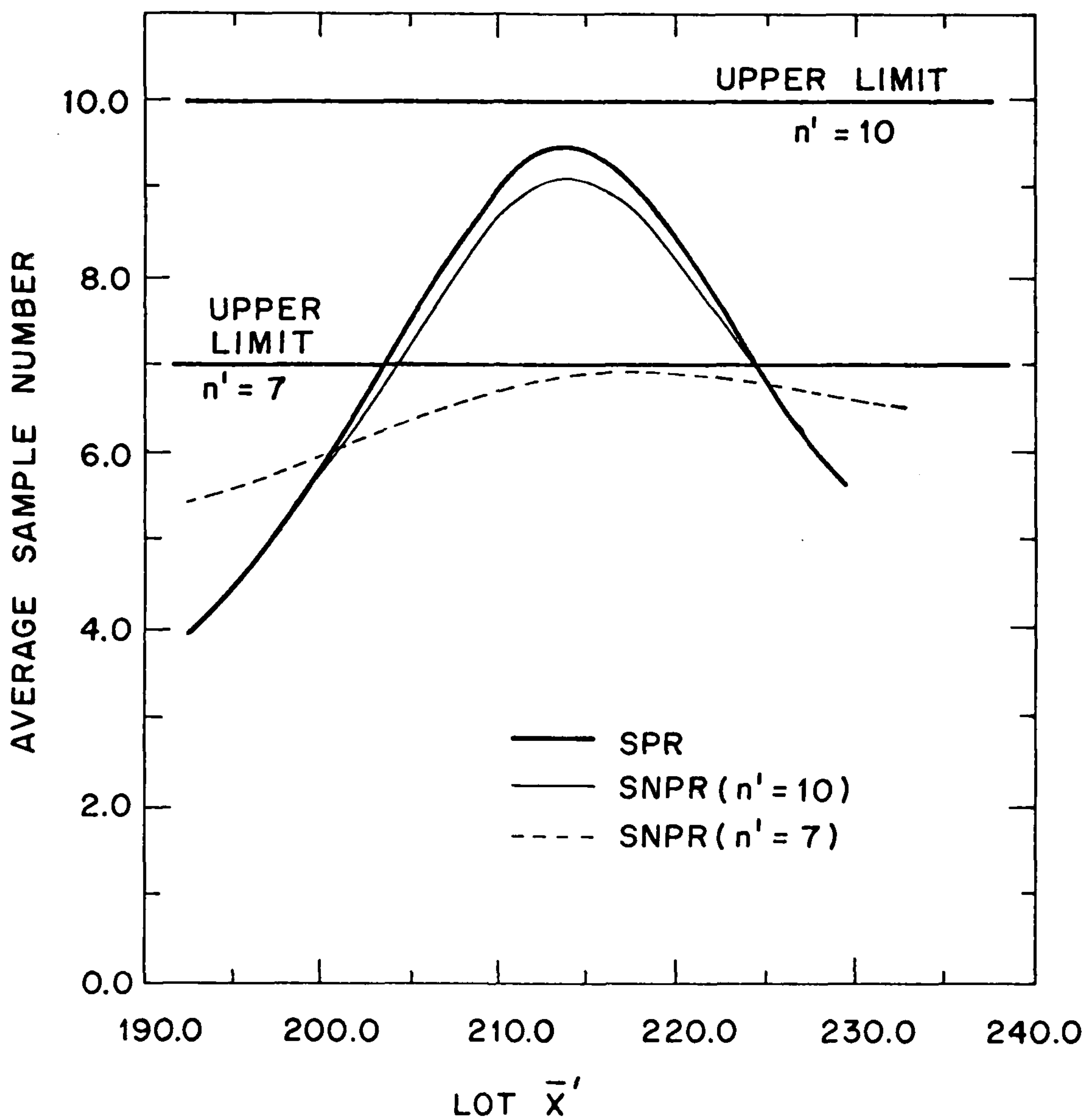

Figure 14. ASN Curves: SNPR Plans (Sample Problem n' = 7, 10) and SPR Plan (Sample Problem)

CHAPTER V

SUMMARY

An SPR plan can materially reduce the amount of required inspection. Studies have shown that the average decrease in sample size is often near 50% when compared with the sample size of a comparable single sampling plan [4]. An SNPR plan can achieve and better the sampling reduction obtained through the use of an SPR plan. The SNPR plan requires no truncation rules and maintains the integrity of its associated OC curve while truncation of the SPR plan makes the plan less discriminating.

Several disadvantages of sequential plans have been noted. The most severe are the cost of administering the plan and the lack of availability of information concerning prevailing levels of quality in each lot. The administrative cost of an SNPR plan can be substantially reduced through the use of the computer code of the heuristic algorithm developed in Chapter III. No bulky tables and complicated charts need be used, only a terminal display. Acceptance and rejection limits would be displayed on a screen in conjunction with input inspection variables. The savings in inspection costs and time could possibly offset any loss due to unavailability of lot quality data. The ultimate extension would be the use of the sampling plan in an automated environment where inspection is performed by machines.

Whatever its use, the SNPR plan for variables inspection is a flexible plan which in many instances outperforms other sampling plans.

Its applications are not limited to one-sided inspection and further research should be able to produce modifications which enable it to be expanded to two-sided inspection (inspection with both upper and lower quality limits). It is hoped that as a result of this research, serious consideration will be given to this plan so as to reap the potential savings that can be a by product of its use.

APPENDIX A

SEQUENTIAL NON-PROBABILITY RATIO (SNPR) AND SEQUENTIAL PROBABILITY RATIO (SPR) TEST PLAN GENERATOR

The following program provides a SNPR test plan, a SPR test plan or both for a given set of input parameters. It also calculates a specified number of points for the associated OC curve(s) of the requested plan(s). Additionally a specified number of points are calculated for the ASN curve of the SPR plan when that plan alone is requested.

The program has three formats:

(1) the SPR test plan;

(2) the SNPR test plan;

(3) the SPR and SNPR test plans.

The following is a brief description of each format's input parameters and output. The input format is "format free," requiring only commas or spaces between input parameters for each line of data. Before input data can be read for any particular format, a numerical format code must be specified on the first data card. The format specification codes are:

CODE	FORMAT
(1)	SPR Test Plan
(2)	SNPR Test Plan
(3)	SPR and SNPR Test Plans

CARD(1): INPUT FORMAT CODE

Format 1: SPR Test Plan

The input format for the SPR test plan requires specification of:

(1) $\bar{X}'_{AQL}$

(2) $\bar{X}'_{LTPD}$

(3) σ'

(4) α (producer's risk)

(5) β (consumer's risk)

(6) NTRUNK

(7) NTERM

$\bar{X}'_{AQL}$ and $\bar{X}'_{LTPD}$ are the means locating a percent defective equal to a desired AQL and LTPD respectively. The program is set up to work with $\bar{X}'_{AQL}$, however, problems where higher values of $\bar{X}'$ are desirable can be accommodated. In this instance, the output must be interpreted accordingly. σ' is the value of the population standard deviation. α is the producer's risk and β the consumer's risk. NTRUNK is the number (n) at which the plan will be terminated. NTERM is the number of points to be plotted for the ASN and OC curves. NTRUNK must be less than or equal to NTERM. A value of 0 input for NTRUNK defaults that value to twice the maximum ASN.

Figure 15 shows an example of an input list and the associated output. In the output listing, a header echoes the input. Following are columns headed by:

N -- sample number

TL -- lower tolerance limit

TU -- upper tolerance limit

```
//DATA.INPUT DD *
1
200,225,30
.05,.10
20,30
        ********************************************
        *                                          *
        * SEQUENTIAL PROBABILITY RATIO SAMPLING PLAN *
        *                                          *
        ********************************************

*****************************************************

        XACL= 200.00  XLTPD= 225.00  SIGMA= 30.00
        ALPHA=0.05  BETA=0.10   TRUNCATE AT X=   20

*****************************************************
*****************************************************

*  N  *     TL          TU      *LCI XBAR*PROB ACC*  ASN  *

*****************************************************
*   1 *    131.45 *    316.55 * 193.75 * 0.9873 *  4.20 *
*   2 *    343.95 *    529.05 * 195.21 * 0.9825 *  4.50 *
*   3 *    556.45 *    741.55 * 196.67 * 0.9757 *  4.84 *
*   4 *    768.95 *    954.05 * 198.12 * 0.9666 *  5.21 *
*   5 *    981.45 *   1166.55 * 199.58 * 0.9542 *  5.62 *
*   6 *   1193.95 *   1379.05 * 201.04 * 0.9377 *  6.07 *
*   7 *   1406.45 *   1591.55 * 202.50 * 0.9159 *  6.55 *
*   8 *   1618.95 *   1804.05 * 203.96 * 0.8877 *  7.05 *
*   9 *   1831.45 *   2016.55 * 205.42 * 0.8518 *  7.57 *
*  10 *   2043.95 *   2229.05 * 206.87 * 0.8075 *  8.07 *
*  11 *   2256.45 *   2441.55 * 208.33 * 0.7543 *  8.54 *
*  12 *   2468.95 *   2654.05 * 209.79 * 0.6928 *  8.93 *
*  13 *   2681.45 *   2866.55 * 211.25 * 0.6244 *  9.22 *
*  14 *   2893.95 *   3079.05 * 212.71 * 0.5516 *  9.38 *
*  15 *   3106.45 *   3291.55 * 214.17 * 0.4774 *  9.41 *
*  16 *   3318.95 *   3504.05 * 215.62 * 0.4051 *  9.30 *
*  17 *   3531.45 *   3716.55 * 217.08 * 0.3375 *  9.07 *
*  18 *   3743.95 *   3929.05 * 218.54 * 0.2766 *  8.75 *
*  19 *   3956.45 *   4141.55 * 220.00 * 0.2235 *  8.36 *
*  20 *   4168.95 *   4354.05 * 221.46 * 0.1786 *  7.93 *
****************************** 222.92 * 0.1414 *  7.48 *
*                            ** 224.37 * 0.1111 *  7.03 *
*                            ** 225.83 * 0.0868 *  6.60 *
*                            ** 227.29 * 0.0675 *  6.19 *
*                            ** 228.75 * 0.0524 *  5.81 *
*                            ** 230.21 * 0.0405 *  5.45 *
*                            ** 231.67 * 0.0313 *  5.13 *
*                            ** 233.12 * 0.0242 *  4.83 *
*                            ** 234.58 * 0.0186 *  4.56 *
*                            ** 236.04 * 0.0143 *  4.31 *
*****************************************************
```

Figure 15. Program Output Listing: Format 1

LOTXBAR -- values of $\bar{X}'$ on x axis of CO and ASN

PROB ACC -- probability of acceptance for $\bar{X}'$

ASN -- average sample number of $\bar{X}'$

The following are the data cards necessary for format 1 input:

CARD (2): $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, σ'

CARD (3): α, β

CARD (4): NTRUNK, NTERM

Format 2: SNPR Test Plan

The input format for the SNPR test plan requires specification of:

(1) $\bar{X}'_{AQL}$

(2) $\bar{X}'_{LTPD}$

(3) σ'

(4) n'

(5) α

(6) β

(7) MAX

(8) NTERM

All but two of the above terms have been described previously. n' is the maximum number of items to be sampled. MAX is the maximum number of iterations to be performed by the heuristic algorithm. This value should initially be set to 5 and adjusted depending on results obtained.

Figure 16 shows an example of an input list and the associated output. In the output listing, a header echoes the input and lists ZA and XABAR the values of Z_{α}, and $\bar{X}'$ calculated by the heuristic

```
//DATA.INPUT DD *
2
200,225,30,10
.05,.10
5,30
**************************************************
*                                                *
* SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN *
*                                                *
**************************************************


**************************************************

     XAQL= 200.00  XLTPD= 225.00  SIGMA= 30.00
     ALPHA=0.05  BETA=0.10   NPRIME=  10
     ZA=  3.01  XABAR= 214.26

**************************************************
**************************************************

*  N  *    TL     *    TU     **LOT XBAR*PROB ACC*

**************************************************
*   1 *    123.97 *    304.54 ** 193.75 * 0.9869 *
*   2 *    348.26 *    508.77 ** 195.21 * 0.9819 *
*   3 *    572.55 *    712.99 ** 196.67 * 0.9752 *
*   4 *    796.83 *    917.21 ** 198.12 * 0.9662 *
*   5 *   1021.12 *   1121.44 ** 199.58 * 0.9541 *
*   6 *   1245.41 *   1325.66 ** 201.04 * 0.9381 *
*   7 *   1469.70 *   1529.89 ** 202.50 * 0.9173 *
*   8 *   1693.98 *   1734.11 ** 203.96 * 0.8907 *
*   9 *   1918.27 *   1938.34 ** 205.42 * 0.8572 *
*  10 *   2142.56 *   2142.56 ** 206.87 * 0.8161 *
********************************* 208.33 * 0.7669 *
*                               ** 209.79 * 0.7099 *
*                               ** 211.25 * 0.6459 *
*                               ** 212.71 * 0.5766 *
*                               ** 214.17 * 0.5045 *
*                               ** 215.62 * 0.4321 *
*                               ** 217.08 * 0.3623 *
*                               ** 218.54 * 0.2976 *
*                               ** 220.00 * 0.2396 *
*                               ** 221.46 * 0.1895 *
*                               ** 222.92 * 0.1474 *
*                               ** 224.37 * 0.1130 *
*                               ** 225.83 * 0.0856 *
*                               ** 227.29 * 0.0641 *
*                               ** 228.75 * 0.0476 *
*                               ** 230.21 * 0.0351 *
*                               ** 231.67 * 0.0258 *
*                               ** 233.12 * 0.0188 *
*                               ** 234.58 * 0.0137 *
*                               ** 236.04 * 0.0099 *
***************************************************
```

Figure 16. Program Output Listing: Format 2

algorithm and used as arguments in the calculation of the SNPR test plan. The column headers are the same as those listed for the SPR test plan.

The following are the data cards necessary for format 2 input:

CARD (2): $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, σ', n'

CARD (3): α, β

CARD (4): MAX, NTERM

Format 3: SPR and SNPR Test Plans

The input format for the SPR and SNPR test plans requires specification of:

(1) $\bar{X}'_{AQL}$

(2) $\bar{X}'_{LTPD}$

(3) σ'

(4) n'

(5) α

(6) β

(7) MAX

(8) NTRUNK

(9) NTERM

Figure 17 shows an example of an input list and the associated output. In the output listing, a header which echoes common and specific input for each test plan. $\bar{X}'$ and Z_{α}, are listed for the SNPR test plan. Column headings are established for n (sample number), upper and lower tolerance limits for both plans, values of $\bar{X}'$ for the

```
//DATA.INPUT DD *
3
200,225,30,10
.05,.10
5,20,30
          ***********************************************************
          *                                                         *
          *  SEQUENTIAL PROBABILITY AND NON-PROBABILITY RATIO SAMPING PLANS  *
          *                                                         *
          ***********************************************************

*******************************************************************************

XACI= 200.00  XITFD= 225.00  SIGMA= 30.00
          SNPR PLAN
ALPHA=0.05  BETA=0.10  NPRIME=  10   ZA=  3.01  XABAR= 214.26
          SPR PLAN
ALPHA=0.05  BETA=0.10   TRUNCATION OCCURS AT N=  20

*******************************************************************************
*******************************************************************************

                  SNPRT            **           SPRT                          SNPRT  *  SPRT
*  N  **      TL      *     TU     **      TL      *     TU     **LCT XBAR*PROB ACC*PROB ACC*

*******************************************************************************
*   1 **     123.97 *    304.54 **     131.45 *    316.55 ** 193.75 * 0.987  * 0.987   *
*   2 **     348.26 *    508.77 **     343.95 *    529.05 ** 195.21 * 0.982  * 0.982   *
*   3 **     572.55 *    712.99 **     556.45 *    741.55 ** 196.67 * 0.975  * 0.976   *
*   4 **     796.83 *    917.21 **     768.95 *    954.05 ** 198.12 * 0.966  * 0.967   *
*   5 **    1021.12 *   1121.44 **     981.45 *   1166.55 ** 199.58 * 0.954  * 0.954   *
*   6 **    1245.41 *   1325.66 **    1193.95 *   1379.05 ** 201.04 * 0.938  * 0.938   *
*   7 **    1469.70 *   1529.89 **    1406.45 *   1591.55 ** 202.50 * 0.917  * 0.916   *
*   8 **    1693.98 *   1734.11 **    1618.95 *   1804.05 ** 203.96 * 0.891  * 0.888   *
*   9 **    1918.27 *   1938.34 **    1831.45 *   2016.55 ** 205.42 * 0.857  * 0.852   *
*  10 **    2142.56 *   2142.56 **    2043.95 *   2229.05 ** 206.87 * 0.816  * 0.808   *
*  11 *************************    2256.45 *   2441.55 ** 208.33 * 0.767  * 0.754   *
*  12 **                       **    2468.95 *   2654.05 ** 209.79 * 0.710  * 0.693   *
*  13 **                       **    2681.45 *   2866.55 ** 211.25 * 0.646  * 0.624   *
*  14 **                       **    2893.95 *   3079.05 ** 212.71 * 0.577  * 0.552   *
*  15 **                       **    3106.45 *   3291.55 ** 214.17 * 0.504  * 0.477   *
*  16 **                       **    3318.95 *   3504.05 ** 215.62 * 0.432  * 0.405   *
*  17 **                       **    3531.45 *   3716.55 ** 217.08 * 0.362  * 0.337   *
*  18 **                       **    3743.95 *   3929.05 ** 218.54 * 0.298  * 0.277   *
*  19 **                       **    3956.45 *   4141.55 ** 220.00 * 0.240  * 0.224   *
*  20 **                       **    4168.95 *   4354.05 ** 221.46 * 0.189  * 0.179   *
************************************************************ 222.92 * 0.147  * 0.141   *
*                                                          ** 224.37 * 0.113  * 0.111   *
*                                                          ** 225.83 * 0.086  * 0.087   *
*                                                          ** 227.29 * 0.064  * 0.068   *
*                                                          ** 228.75 * 0.048  * 0.052   *
*                                                          ** 230.21 * 0.035  * 0.041   *
*                                                          ** 231.67 * 0.026  * 0.031   *
*                                                          ** 233.12 * 0.019  * 0.024   *
*                                                          ** 234.58 * 0.014  * 0.019   *
*                                                          ** 236.04 * 0.010  * 0.014   *
*******************************************************************************
```

Figure 17. Program Output Listing: Format 3

OC curves and the associated probabilities of acceptance for each plan.

The following are the data cards necessary for format 3 input:

CARD (2): $\bar{X}'_{AQL}$, $\bar{X}'_{LTPD}$, σ', n'

CARD (3): α, β

CARD (4): MAX, NTRUNK, NTERM

Program Listing

Following is a listing of the computer code which generates the test plans. It consists of six subroutines and a driver routine. Each routine is commented with a description and other documentation.

```
*** 20-C (03/04/81--1844)

C                        MAIN PRCGRAM
C                        ************
C THIS IS THE DRIVER ROUTINE FCR THE PRCGRAM.  IT READS DATA AND CALLS
C    INDIVIDUAL SUBROUTINES BASED ON A PRESCRIBED FCRMAT, IPRINT.
C    THERE ARE THREE POSSIBLE FORMATS AS DESCRIBED IN THE USERS GUIDE.
C    THE SPR TEST PLAN FORMAT CALLS SUBRCUTINE SPRT WHERE THE TOLERANCE
C    LIMITS FOR THE PLAN ARE CALCULATED.  IT THEN CALLS SUBRCUTINE
C    OUTPUT WHERE THE CALCULATICNS ARE OUTPUT ACCORDING TO THE SPECIFIED
C    FORMAT.  THE SNPR TEST PLAN FCRMAT CALLS SUBROUTINE SPRT TO
C    OBTAIN THE TOLERANCE LIMITS FCR N=1 IN THE SPR TEST PLAN.
C    NEXT, SUBROUTINE SSP (SINGLE SAMPLING PLAN) CALCULATES THE
C    SEED VALUES OF ZA AND XA USED BY THE NEXT CALLED SUBROUTINE,
C    SUBROUTINE SEARCH.  THIS SUBRCUTINE RETURNS THE UPPER AND LCWER
C    TOLERANCE LIMITS FCR THE SNPR TEST PLAN.  SUBROUTINE OUTPUT IS THEN
C    CALLED TO PRODUCE THE REQUIRED OUTPUT LISTING.  THE COMBINED SPR
C    AND SNPR TEST PLAN FORMAT IS THE SAME AS THAT OF FCRMAT 2 EXCEPT
C    THE OUTPUT FORMAT IS MODIFIED TC HANDLE BOTH TEST PLAN FORMATS.
C
C**********************************************************************
      CCMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPR,XLTPD,ZASNPR,
     *       ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),CCSNPR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
     *       SPRTU(500),XPRICT(100)
      READ,IPRINT
      GCTO(1,2,4),IPRINT
C  PROCESSING FORMAT FCR SPRT PLAN.
1     READ,XAQL,XLTPD,SIGMA
      READ,ALPHA,BETA
      READ,NTRUNK,NTERM
      CALL SPRT
      CALL OUTPUT
      GOTO 5
C  PROCESSING FORMAT FCR SNPRT PLAN.
2     READ,XAQL,XLTPD,SIGMA,NPRIME
      READ,ALPHA,BETA
      READ,MAX,NTERM
C ONLY THE TOLERANCE LIMITS AT N=1 ARE NEEDED FOR THIS FCRMAT.
      NTRUNK=1
3     CALL SPRT
      CALL SSP
      CALL SEARCH
      CALL SNPROC
      CALL OUTPUT
      GCTO 5
C  PROCESSING FORMAT FCR SPRT AND SNPRT PLANS
4     READ,XAQL,XLTPD,SIGMA,NPRIME
      READ,ALPHA,BETA
      READ,MAX,NTRUNK,NTERM
      GCTO 3
5     STCP
      END
```

```
                    SUBROUTINE SSP
C                   **************
C THIS SUBROUTINE IS A FEEDER ROUTINE TO SUBROUTINE SEARCH.  IT CALCU-
C    LATES THE SEED VALUES OF ZALPHA USING THE VALUES OF TL(1) AND TU(1)
C    (SEE CHAPTER III OF THESIS) GENERATED IN SUBROUTINE SPRT.  IT ALSO
C    CALCULATES THE SEED VALUE OF XBARPRIME FROM XABAR, THE INDIFFERENCE
C    POINT OF ACCEPTANCE LIMIT, FOR A SINGLE SAMPLING PLAN.
C
C*********************************************************************
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNFR,XLTPD,ZASNPF,
     *       ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),OCSNFR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
     *       SPRTU(500),XERICT(100)
C CALCULATE THE SEED VALUE OF ZALPHA, ZASPRT.
      ZASPRT=(SPRTU(1)-SPRTL(1))/2/SIGMA
C CALCULATE THE REQUIRED SAMPLE NUMBER OF THE SINGLE SAMPLING PLAN
C    RNSSP AND THE VALUE OF THE NEXT HIGHEST INTEGER, NSSP.
      RNSSP=((ZPRB(ALPHA)+ZPRB(BETA))*SIGMA/ABS(XAQL-XLTPD))**2
      NSSP=RNSSP
C AFTER DETERMINING WHICH VALUE IS GREATER, XAQL AND XLTPD, CALCULATE
C    THE INDIFFERENCE POINT WITH N=RNSSP AND N=NSSP. TAKE THE AVERAGE
C    OF THESE TWO VALUES, XA, AND USE IT AS THE SEED, XBARPRIME.
      IF(XAQL.GE.XLTPD) GOTO 10
      XABAR=(ZPRB(ALPHA)*SIGMA/RNSSP**0.5)+XAQL
      XABI=(ZPRB(ALPHA)*SIGMA/NSSP**0.5)+XAQL
      XA=AMAX1(XABAR,XABI)-ABS(XABAR-XABI)/2
      GOTO 20
10    XABAR=(-ZPRB(ALPHA)*SIGMA/RNSSP**0.5)+XAQL
      XABI=(-ZPRB(ALPHA)*SIGMA/NSSP**0.5)+XAQL
      XA=AMAX1(XABAR,XABI)-ABS(XABAR-XABI)/2
20    RETURN
      END
```

```
              SUBROUTINE SPRT
C             ***************
C THIS ROUTINE USES THE EQUATIONS DEVELOPED BY WALD (SEE CHAPTER II
C    OF THESIS) FOR THE SPR TEST.  IT GENERATES A TEST PLAN FOR THE
C    SPECIFIED INPUT VALUES OF XAQL, XLTPD,SIGMA, ALPHA AND BETA.  IT
C    TRUNCATES THE PLAN AT THE INPUT VALUE OF NTRUNK OR THE DEFAULT
C    VALUE WHICH IS TWICE THE VALUE OF THE MAXIMUM ASN AT XAQL OR
C    XLTPD.  VALUES FOR THE OPERATING CHARACTERISTIC (OC) AND AVERAGE
C    SAMPLE NUMBER (ASN) CURVES ARE ALSO GENERATED.
C
C*********************************************************************
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPR,XLTPD,ZASNPP,
     *       ZASPRT,IPRINT,MAX,NMAX,NREPRE,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),OCSNPR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
     *       SPRTU(500),XBRLCT(100)
C DECLARATION OF FUNCTION STATEMENTS.
      ACC(N)=H0+N*S;REJ(N)=H1+N*S
      H(THETA)=(XAQL+XLTPD-2*THETA)/(XLTPD-XAQL)
      PACC(THETA)=(A**H(THETA)-1)/(A**H(THETA)-B**H(THETA))
      ASN(THETA)=(H1+PACC(THETA)*(H0-H1))/(THETA-S)
C CALCULATION OF EQUATION PARAMETERS.
      A=(1-BETA)/ALPHA
      B=BETA/(1-ALPHA)
      S=(XAQL+XLTPD)/2
      H0=SIGMA**2*ALOG(B)/(XLTPD-XAQL)
      H1=SIGMA**2*ALOG(A)/(XLTPD-XAQL)
C DETERMINATION OF TRUNCATION VALUE.
      ITERM=NTRUNK
      IF (NTRUNK.LT.0.5) ITERM=2*AMAX1(ASN(XAQL),ASN(XLTPD))
      NTRUNK=ITERM
C CALCULATION OF THE TOLERANCE LIMITS.
      DO 100 I=1,ITERM
         SPRTL(I)=ACC(I)
100      SPRTU(I)=REJ(I)
C CALCULATION OF THE STEP FACTOR USED FOR THE XBARPRIME OF THE OC
C    AND ASN CURVES. THE LIMITS FOR XBARPRIME EXTEND BEYOND XAQL AND
C    XLTPD BY AN AMOUNT DETERMINED BY THE VALUES OF ALPHA AND BETA.
      FACT1=ALPHA*5;FACT2=BETA*5;FACTOR=(1+FACT1+FACT2)/NTERM
      RINCRM=0;XLTPD1=XAQL-FACT1*ABS(XAQL-XLTPD);NMAX=0
C THIS SEGMENT FILLS THE ARRAY XBRLCT WITH THE VALUES OF THE ARGUMENTS
C    FOR CALCULATION OF THE OC AND ASN CURVES.  ARRAYS OC AND ASN ARE
C    FILLED WITH THE ASSOCIATED VALUES OF THE PROBABILITY OF ACCEPTANCE
C    AND ASN.
      DO 200 I=1,NTERM
         XBRLCT(I)=XLTPD1+RINCRM
         OC(I)=PACC(XLTPD1+RINCRM)
         ASNCRV(I)=ASN(XLTPD1+RINCRM)
C CALCULATION OF THE MAXIMUM ASN.
         IF(ASNCRV(I).GT.NMAX) NMAX=ASNCRV(I)+1
200      RINCRM=RINCRM+FACTOR*ABS(XAQL-XLTPD)
      RETURN
      END
```

```
                          SUBROUTINE SNPRT(ZALPHA,XBAR)
C                         *****************************
C THIS SUBROUTINE USES THE INPUT VALUES OF XAQL, XLTPD, SIGMA AND NPRIME
C    , ALSO THE CALLING ARGUMENTS ZALPHA AND XBAR TO GENERATE AN SNPP
C    TEST PLAN WITH ASSOCIATED ALPHA(ASUM) AND BETA(BSUM).
C*********************************************************************
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPR,XLTPD,ZASNPP,
     *       ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),OCSNPP(100),SNPRTI(500),SNPPTU(500),SPRTL(500),
     *       SPRTU(500),XERLCT(100)
C  CALCULATE THE STEP INTERVAL (SEE CHAPTER III OF THESIS).
      ENTRVL=ZALPHA*SIGMA/(NPRIME-1)
C  CALCULATE THE CONTROL LIMITS AND THE ALPHA AND BETA OF THE TEST AND
C    STORE THE RESULTS IN THE ARRAYS SNPRTU AND SNPRTL.
      DO 12  J=1,NPRIME
          RJ=J
          SNPRTU(J)=XBAR*J+((NPRIME-J)*ENTRVL)
          SNPRTL(J)=XBAR*J-((NPRIME-J)*ENTRVL)
C  CALCULATE ALPHA USING XAQL.
          DEVIAT=(SNPRTU(J)-J*XAQL)/(SQRT(RJ)*SIGMA)
          IF(DEVIAT.GT.0.0)GOTO 1
          ALPH=1-PRBZ(-DEVIAT)
          GOTO 2
1         ALPH=PRBZ(DEVIAT)
C  CALCULATE BETA USING XAQL.
2         DEVIBT=(SNPRTL(J)-J*XAQL)/(SQRT(RJ)*SIGMA)
          IF (DEVIBT.GT.0.0) GOTO 3
          BET=PRBZ(-DEVIBT)
          GOTO 4
3         BET=1-PRBZ(DEVIBT)
C  CALCULATE ALPHA SUM.
4         CONTINUE
          IF(J.GT.1.5)GOTO 5
          ASUM=ALPH
          PROD=1-ALPH-BET
          GOTO 6
5         ASUM=ASUM+ALPH*PROD
          PROD=PROD*(1-ALPH-BET)
C  CALCULATE ALPHA USING XLTPD.
6         DEVIAT=(SNPRTU(J)-J*XLTPD)/(SQRT(RJ)*SIGMA)
          IF(DEVIAT.GT.0.0)GOTO 7
          ALPH=1-PRBZ(-DEVIAT)
          GOTO 8
7         ALPH=PRBZ(DEVIAT)
C  CALCULATE BETA USING XLTPD.
8         DEVIBT=(SNPRTI(J)-J*XLTPD)/(SQRT(RJ)*SIGMA)
          IF(DEVIBT.GT.0.0)GOTO 9
          BET=PRBZ(-DEVIBT)
          GOTO 10
9         BET=1-PRBZ(DEVIBT)
C  CALCULATE BETA SUM.
10        IF(J.GT.1.5)GOTO 11
          BSUM=BET
          PRODB=1-ALPH-BET
          GOTO 12
11        BSUM=BSUM+BET*PRODB
          PRODB=PRODB*(1-ALPH-BET)
12        CONTINUE
      RETURN
      END
```

```
                  SUBROUTINE SNPROC
C                 *****************
C THIS SUBROUTINE CALCULATES VALUES FOR THE OC CURVE OF AN SNPR TEST
C    PLAN.
C
C*************************************************************************
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPP,XLTPD,ZASNPR,
     *       ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),OCSNPR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
     *       SPRTU(500),XBPLOT(100)
C  CALCULATE VALUES FOR THE OC CURVE USING THE ARRAY XBPLOT(100).
      DO 8 K=1,NTERM
C  CALCULATE THE PROBABILITY OF ACCEPTANCE FOR A SPECIFIC XBPLOT.
      DO 7 J=1,NPRIME
         RJ=J
C  CALCULATE ALPHA USING XBPLOT (K).
1        DEVIAT=(SNPRTU(J)-J*XBPLOT(K))/(SQRT(RJ)*SIGMA)
         IF(DEVIAT.GT.0.0)GOTO 2
         ALPH=1-PRBZ(-DEVIAT)
         GOTO 3
2        ALPH=PRBZ(DEVIAT)
C  CALCULATE BETA USING XBPLOT(K).
3        DEVIBT=(SNPRTL(J)-J*XBPLOT(K))/(SQRT(RJ)*SIGMA)
         IF(DEVIBT.GT.0.0)GOTO 4
         BET=PRBZ(-DEVIBT)
         GOTO 5
4        BET=1-PRBZ(DEVIBT)
C  CALCULATE BET SUM.
5        IF(J.GT.1.5)GOTO 6
         BETSUM=BET
         PRODB=1-ALPH-BET
         GOTO 7
6        BETSUM=BETSUM+BET*PRODB
         PRODB=PRODB*(1-ALPH-BET)
7        CONTINUE
8     OCSNPR(K)=BETSUM
      RETURN
      END
```

```
                          SUBROUTINE SEAFCH
C                         ****************
C THIS SUBROUTINE IN CCNJUNCTICN WITH THE CTHER SUEROUTINES FINDS THE
C    PARAMETERS ZALPHA AND XBARPRIME WHICH FOR A SPECIFIC VALUE OF
C    NPRIME YIELD AN SNPR TEST PLAN WITH ASUM=ALPHA AND BSUM=BETA.
C    THIS IS DONE USING THE HEURISTIC ALGCFITHM DEVELOPED IN CHAPTER
C    III.  USING THE SEED VALUES OF ZAIPHA AND XBARPRIME FRCM SUBRCUTINE
C    SSP, A MODIFIED VALUE OF XBARPRIME (XBARPRIME(NPRIME)) IS FOUND.
C    AFTER A CHECK TC SEE IF THE VALUES CF ASUM AND BSUM ARE WITHIN THE
C    DESIRED TOLERANCE, THE SUBROUTINE IS TERMINATED (TCLERANCE MET) CR
C    THE VALUES OF ZALPHA AND XBARPRIME ARE MODIFIED (TOLERANCE NOT
C    MET).  A RECALCULATICN OF ASUM AND BSUM WITH THE MODIFIED CALLING
C    ARGUMENTS IS FOLLOWED BY ANOTHER TOLERANCE CHECK.  THE PROCESS
C    CONTINUES UNTIL THE VALUES CF ASUM AND BSUM ARE WITHIN TOLERANCE CR
C    THE MAXIMUM NUMBER OF ITERATICNS (MAX) HAVE BEEN PERFORMED.
C
C**********************************************************************
      CCMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPR,XLTPD,ZASNPR,
     *       ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),CCSNPR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
     *       SPRTU(500),XBBLCT(100)
      RNPRIM=NPRIME;RNMAX=NMAX
C CALCULATE ZALPHA(NPRIME)
      ZAT=ZASPRT*(1.0+(1.0-RNPRIM/RNMAX));XBRTMP=XA;NTIME=0
C  CHECK TO SEE IF ASUM AND BSUM ARE WITHIN 0.001 CF THE SPECIFIED
C    ALPHA AND BETA AFTER CALLING SUBRCUTINE SNPRT.
1     CALL SNPRT(ZAT,XBRTMP)
      IF(ABS(ALPHA-ASUM).LE.0.001.AND.ABS(BETA-BSUM).LE.0.001)GOTO 50
      IF(NTIME.GT.MAX) GO TO 50
C  CALCULATE THE DISTANCES OF ASUM AND BSUM FRCM ALPHA AND BETA.
      DIF=ABS((ASUM-ALPHA)-(BSUM-BETA))
C  DETERMINE IF ASUM MUST INCREASE OR DECREASE AND ALTER ACCORDINGLY.
C    CALCULATE NEW ASUM AND BSUM.
      IF((ASUM-ALPHA).LT.(BSUM-BETA))GOTO 10
      SIGN=1
      GOTO 20
10    SIGN=-1
20    AMARK=DIF*(1/(BETA/ALPHA+1))
C ALTER XBARPRIME BY A FIXED AMOUNT.
      SENEST=SIGMA/100.0
      XBRTM1=XBRTMP+SENEST*SIGN;ASUMPV=ASUM
C CALCULATE THE EFFECT CF THE ALTERATICN.
      CALL SNPRT(ZAT,XBRTM1)
C CALCULATE THE PROPORTIONATE  VALUE BY WHICH XBARPRIME MUST BE ALTERED
C    TO MEET THE OBJECTIVE VALUE OF ALPHA.
      SENCAL=SENEST*AMARK/ABS(ASUMPV-ASUM)
      XBRTMP=XBRTMP+SENCAL*SIGN
      CALL SNPRT(ZAT,XBRTMP)
C  CHECK TO SEE IF ASUM AND BSUM ARE WITIN LIMITS.
      IF(ABS(ALPHA-ASUM).LE.0.001.AND.ABS(BETA-BSUM).LE.0.001)GOTO 50
C  DETERMINE IF ASUM MUST INCREASE OR DECREASE.  ALTER ZAT ACCORDINGLY
C    FOR A NEW ASUM AND BSUM.
      IF((ASUM-ALPHA).LT.0.0)GCTC 30
      SIGN=1
      GOTO 40
30    SIGN=-1
40    AMARK=ABS(ASUM-ALPHA)
C ALTER ZALPHA BY A FIXED AMCUNT.
      ZATT=ZAT+0.01*SIGN;ASUMPV=ASUM
      CALL SNPRT(ZATT,XBRTMP)
```

```
C CALCULATE THE PROPORTIONATE  CHANGE IN ZALPHA NECESSARY TO ACHIEVE THE
C     OBJECTIVE VALUES OF ALPHA AND BETA.
      SENCAL=0.01*AMARK/ABS(ASOMPV-ASOM)
      ZAT=ZAT+SENCAL*SIGN
C INCREMENT THE ITERATION COUNTER.
      NTIME=NTIME+1
      GOTO 1
50    ZASNPP=ZAT;XASNPR=XERTMP
      RETURN
      END
```

```
                       SUBROUTINE OUTPUT
C                      *****************
C THIS SUBROUTINE FORMATS AND OUTPUTS THE RESULTS ACCORDING TO THE
C    REQUESTED FORMAT.
C
C*********************************************************************
      COMMON ALPHA,ASUM,BETA,BSUM,SIGMA,XA,XAQL,XASNPR,XLTPD,ZASNPR,
     *       ZASPRT,IPRINT,MAX,NMAX,NPRIME,NTERM,NTRUNK,ASNCRV(100)
     *       ,OC(100),CCSNPR(100),SNPRTL(500),SNPRTU(500),SPRTL(500),
     *       SPRTU(500),XERLCT(100)
C  DETERMINE THE OUTPUT FORMAT.
      ITERM=NTRUNK
      IF(IPRINT.EQ.1)GOTO 1
      IF(IPRINT.EQ.2)GOTO 2
      IF(IPRINT.EQ.3)GOTO 3
C  FORMAT FOR THE SPR PLAN.
1     WRITE(6,100)
      WRITE(6,101)
      WRITE(6,102)
      WRITE(6,101)
      WRITE(6,100)
      PRINT,' '
      PRINT,' '
      WRITE(6,103)
      PRINT,' '
      WRITE(6,104) XAQL,XLTPD,SIGMA
      WRITE(6,105) ALPHA,BETA,ITERM
      PRINT,' '
      WRITE(6,103)
      WRITE(6,103)
      PRINT,' '
      WRITE(6,106)
      PRINT,' '
      WRITE(6,103)
      DO 10 I=1,ITERM
10    WRITE(6,107) I,SPRTL(I),SPRTU(I),XERLCT(I),OC(I),ASNCRV(I)
      WRITE(6,108) XERLCT(ITERM+1),OC(ITERM+1),ASNCRV(ITERM+1)
      ITEMP1=ITERM+2
      DO 11 I=ITEMP1,NTERM
11    WRITE(6,109) XERLOT(I),OC(I),ASNCRV(I)
      WRITE(6,103)
      GOTO 1000
C  FORMAT FOR THE SNPR PLAN.
2     WRITE(6,200)
      WRITE(6,201)
      WRITE(6,202)
      WRITE(6,201)
      WRITE(6,200)
      PRINT,' '
      PRINT,' '
      WRITE(6,200)
      PRINT,' '
      WRITE(6,104) XAQL,XLTPD,SIGMA
      WRITE(6,204) ASUM,BSUM,NPRIME
      WRITE(6,203) ZASNPR,XASNPR
      PRINT,' '
      WRITE(6,200)
      WRITE(6,200)
      PRINT,' '
      WRITE(6,205)
```

```
      PRINT,' '
      WRITE(6,200)
      DO 20 I=1,NPRIME
20    WRITE(6,206) I,SNPRTL(I),SNPRTU(I),XBRLOT(I),OCSNPR(I)
      WRITE(6,207) XBRLOT(NPRIME+1),OCSNPR(NPRIME+1)
      ITEMP1=NPRIME+2
      DO 21 I=ITEMP1,NTERM
21    WRITE(6,208) XBRLOT(I),OCSNPR(I)
      WRITE(6,200)
      GOTO 1000
C  FORMAT FOR THE SNPR AND SPR PLANS.
3     WRITE(6,300)
      WRITE(6,301)
      WRITE(6,302)
      WRITE(6,301)
      WRITE(6,300)
      PRINT,' '
      PRINT,' '
      WRITE(6,303)
      PRINT,' '
      WRITE(6,304) XAQL,XLTPD,SIGMA
      WRITE(6,305)
      WRITE(6,306) ASUM,BSUM,NPRIME,ZASNPR,XASNPR
      WRITE(6,307)
      WRITE(6,308) ALPHA,BETA,ITERM
      PRINT,' '
      WRITE(6,303)
      WRITE(6,303)
      PRINT,' '
      WRITE(6,3091)
      WRITE(6,309)
      PRINT,' '
      WRITE(6,303)
      DO 30 I=1,NPRIME
30    WRITE(6,310) I,SNPRTL(I),SNPRTU(I),SPRTL(I),SPRTU(I),XBRLOT(I),OCS
     *NPR(I),OC(I)
      I=NPRIME+1
      WRITE(6,311) I,SPRTL(I),SPRTU(I),XBRLOT(I),OCSNPR(I),OC(I)
      ITEMP=NPRIME+2
      DO 31 I=ITEMP,ITERM
31    WRITE(6,312) I,SPRTL(I),SPRTU(I),XBRLOT(I),OCSNPR(I),OC(I)
      WRITE(6,313) XBRLOT(ITERM+1),OCSNPR(ITERM+1),OC(ITERM+1)
      ITEMP1=ITERM+2
      DO 32 I=ITEMP1,NTERM
32    WRITE(6,314) XBRLOT(I),OCSNPR(I),OC(I)
      WRITE(6,303)
      GOTO 1000
100   FORMAT(6X,46('*'))
101   FORMAT(6X,'*',44X,'*')
102   FORMAT(6X,'* SEQUENTIAL PROBABILITY RATIO SAMPLING PLAN *')
103   FORMAT(1X,56('*'))
104   FORMAT(6X,'XAQL=',F7.2,2X,'XLTPD=',F7.2,2X,'SIGMA=',F6.2)
105   FORMAT(6X,'ALPHA=',F4.2,2X,'BETA=',F4.2,3X,'TRUNCATE AT X= ',I4)
106   FORMAT(1X,'*',2X,'N',2X,'*',4X,'TL',8X,' TU',4X,'*LOT XBAR*PROB AC
     *C*  ASN  *')
107   FORMAT(1X,'*',I4,1X,'*',F9.2,' *',F9.2,' *',F7.2,' * ',F6.4,' *',
     *F6.2,' *')
108   FORMAT(1X,29('*'),F7.2,' * ',F6.4,' *',F6.2,' *')
109   FORMAT(1X,'*',26X,'**',F7.2,' * ',F6.4,' *',F6.2,' *')
200   FORMAT(1X,50('*'))
```

```
201   FORMAT(1X,'*',48X,'*')
202   FORMAT(1X,'* SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN *')
203   FORMAT(6X,'ZA=',F6.2,2X,'XABAR=',F7.2)
204   FORMAT(6X,'ALPHA=',F4.2,2X,'BETA=',F4.2,3X,'NPRIME=',I4)
205   FORMAT(1X,'*   N   *',4X,'TL',4X,'*',4X,'TU',4X,'**ICT XBAR*PROB ACC
     **')
206   FORMAT(1X,'*',I4,' *',F9.2,' *',F9.2,' **',F7.2,' * ',F6.4,' *')
207   FORMAT(1X,30('*'),F7.2,' * ',F6.4,' *')
208   FORMAT(1X,'*',27X,'**',F7.2,' * ',F6.4,' *')
300   FORMAT(8X,68('*'))
301   FORMAT(8X,'*',66X,'*')
302   FORMAT(8X,'*  SEQUENTIAL PROBABILITY AND NON-PROBABILITY RATIO SAM
     *PING PLANS  *')
303   FORMAT(1X,82('*'))
304   FORMAT(1X,'XAQL=',F7.2,2X,'XLTPD=',F7.2,2X,'SIGMA=',F6.2)
305   FORMAT(10X,'SNPR PLAN')
306   FORMAT(1X,'ALPHA=',F4.2,2X,'BETA=',F4.2,2X,'NPRIME=',I4,3X,'ZA=',F
     *6.2,2X,'XABAR=',F7.2)
307   FORMAT(10X,'SPR PLAN')
308   FORMAT(1X,'ALPHA=',F4.2,2X,'BETA=',F4.2,3X,'TRUNCATION OCCURS AT N
     *=',I4)
3091  FORMAT(17X,'SNPRT',8X,'**',9X,'SPRT',20X,'SNPRT',2X,'*  SPRT')
309   FORMAT(1X,'*   N  **',4X,'TL',4X,'*',4X,'TU',4X,'**',4X,'TL',4X,'*'
     *,4X,'TU',4X,'**ICT XBAR*PROB ACC*PROB ACC*')
310   FORMAT(1X,'*',I4,' **',F9.2,' *',F9.2,' **',F9.2,' *',F9.2,' **',F
     *7.2,' * ',F5.3,'  * ',F5.3,'   *')
311   FORMAT(1X,'*',I4,1X ,25('*'),F9.2,' *',F9.2,' **',F7.2,' * ',F5.3,
     *'  * ',F5.3,'   *')
312   FORMAT(1X,'*',I4,' **',21X,'**',F9.2,' *',F9.2,' **',F7.2,' * ',F5
     *.3,'  * ',F5.3,'   *')
313   FORMAT(1X,54('*'),F7.2,' * ',F5.3,'  * ',F5.3,'   *')
314   FORMAT(1X,'*',51X,'**',F7.2,' * ',F5.3,'  * ',F5.3,'   *')
1000  RETURN
```

APPENDIX B

SELECT SNPR TEST PLANS

The following pages contain SNPR Test Plans for twelve sets of sample data. In each case $\bar{X}'_{AQL}$ = 45 and $\bar{X}'_{LTPD}$ = 60. Three different values are used for σ'; 15, 22.5 and 30 (these values represent 1, 1.5 and 2 times the difference ($\bar{X}'_{AQL}$ - $\bar{X}'_{LTPD}$). For each value of σ', two plans are generated with n' equal to the maximum ASN of the corresponding SPR test and approximately 70% of that value. Two sets of plans for α = .05, β = .10 and α = .01, β = .01 are generated in each of the above cases.

$\bar{X}'_{AQL}$	$\bar{X}'_{LTPD}$	α	β	σ'	n'
40	65	.05	.10	15.0	6
					7
				22.5	11
					15
				30.0	19
					27
		.01	.01	15.0	15
					22
				22.5	34
					48
				30.0	60
					85

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 30.00
ALPHA=0.01 BETA=0.01 NPRIME= 85
ZA= 9.19 XABAR= 52.51

N	TL	TU	LCT MEAN	PROB ACC
1	-223.19	328.22	44.25	0.9936
2	-167.40	377.45	44.43	0.9928
3	-111.60	426.68	44.62	0.9920
4	-55.81	475.91	44.80	0.9910
5	-0.01	525.15	44.98	0.9899
6	55.79	574.38	45.17	0.9887
7	111.58	623.61	45.35	0.9874
8	167.38	672.84	45.53	0.9859
9	223.17	722.07	45.72	0.9842
10	278.97	771.30	45.90	0.9824
11	334.76	820.53	46.08	0.9803
12	390.56	869.76	46.27	0.9780
13	446.36	919.00	46.45	0.9754
14	502.15	968.23	46.63	0.9725
15	557.95	1017.46	46.82	0.9694
16	613.74	1066.69	47.00	0.9658
17	669.54	1115.92	47.18	0.9619
18	725.33	1165.15	47.37	0.9576
19	781.13	1214.38	47.55	0.9527
20	836.93	1263.62	47.73	0.9474
21	892.72	1312.85	47.92	0.9415
22	948.52	1362.08	48.10	0.9350
23	1004.31	1411.31	48.28	0.9279
24	1060.11	1460.54	48.47	0.9200
25	1115.91	1509.77	48.65	0.9114
26	1171.70	1559.00	48.83	0.9019
27	1227.50	1608.23	49.02	0.8916
28	1283.29	1657.47	49.20	0.8803
29	1339.09	1706.70	49.38	0.8681
30	1394.88	1755.93	49.57	0.8548
31	1450.68	1805.16	49.75	0.8405
32	1506.48	1854.39	49.93	0.8250
33	1562.27	1903.62	50.12	0.8084
34	1618.07	1952.85	50.30	0.7907
35	[illegible]	[illegible]	50.48	0.7718
36	1729.66	2051.32	50.67	0.7519
37	1785.45	2100.55	50.85	0.7306
38	1841.25	2149.78	51.03	0.7084
39	1897.05	2199.01	51.22	0.6851
40	1952.84	2248.24	51.40	0.6609
41	2008.64	2297.47	51.58	0.6358
42	2064.43	2346.70	51.77	0.6102
43	2120.23	2395.94	51.95	0.5836
44	2176.02	2445.17	52.13	0.5567
45	2231.82	2494.40	52.32	0.5294
46	2287.62	2543.63	52.50	0.5023
47	2343.41	2592.86	52.68	0.4746
48	2399.21	2642.09	52.87	0.4473
49	2455.00	2691.32	53.05	0.4204
50	2510.80	2740.56	53.23	0.3939
51	2566.60	2789.79	53.42	0.3679
52	2622.39	2839.02	53.60	0.3428
53	2678.19	2888.25	53.78	0.3184
54	2733.98	2937.48	53.97	0.2950
55	2789.78	2986.71	54.15	0.2726
56	2845.57	3035.94	54.33	0.2513
57	2901.37	3085.17	54.52	0.2311
58	2957.17	3134.41	54.70	0.2120
59	3012.96	3183.64	54.88	0.1941
60	3068.76	3232.87	55.07	0.1773
61	3124.55	3282.10	55.25	0.1617
62	3180.35	3331.33	55.43	0.1472
63	3236.14	3380.56	55.62	0.1338
64	3291.94	3429.79	55.80	0.1214
65	3347.74	3479.02	55.98	0.1100
66	3403.53	3528.26	56.17	0.0996
67	3459.33	3577.49	56.35	0.0900
68	3515.12	3626.72	56.53	0.0812
69	3570.92	3675.95	56.72	0.0732
70	3626.72	3725.18	56.90	0.0660
71	3682.51	3774.41	57.08	0.0594
72	3738.31	3823.64	57.27	0.0534
73	3794.10	3872.88	57.45	0.0480
74	3849.90	3922.11	57.63	0.0431
75	3905.69	3971.34	57.82	0.0387
76	3961.49	4020.57	58.00	0.0347
77	4017.29	4069.80	58.18	0.0311
78	4073.08	4119.03	58.37	0.0279
79	4128.87	4168.26	58.55	0.0250
80	4184.67	4217.49	58.73	0.0224
81	4240.46	4266.72	58.92	0.0200
82	4296.26	4315.95	59.10	0.0179
83	4352.05	4365.18	59.28	0.0160
84	4407.85	4414.42	59.47	0.0143
85	4463.65	4463.65	59.65	0.0128

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 15.00
ALPHA=0.05 BETA=0.10 NPRIME= 7
ZA= 2.42 XABAR= 53.53

N	TL	TU	LOT XBAR	PROB ACC
1	17.23	89.83	41.25	0.9870
2	76.82	137.32	42.12	0.9819
3	136.40	184.80	43.00	0.9751
4	195.99	232.28	43.87	0.9659
5	255.57	279.77	44.75	0.9536
6	315.15	327.25	45.62	0.9373
7	374.74	374.74	46.50	0.9162
			47.37	0.8891
			48.25	0.8551
			49.12	0.8136
			50.00	0.7641
			50.87	0.7070
			51.75	0.6432
			52.62	0.5744
			53.50	0.5028
			54.37	0.4311
			55.25	0.3620
			56.12	0.2977
			57.00	0.2400
			57.87	0.1900
			58.75	0.1478
			59.62	0.1133
			60.50	0.0857
			61.37	0.0641
			62.25	0.0475
			63.12	0.0349
			64.00	0.0255
			64.87	0.0185
			65.75	0.0134
			66.62	0.0096

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 15.00
ALPHA=0.05 BETA=0.10 NPRIME= 6
ZA= 2.81 XABAR= 53.52

N	TL	TU	LOT XBAR	PROB ACC
1	11.30	95.73	41.25	0.9881
2	73.27	140.81	42.12	0.9831
3	135.23	185.89	43.00	0.9762
4	197.19	230.96	43.87	0.9669
5	259.16	276.04	44.75	0.9542
6	321.12	321.12	45.62	0.9375
			46.50	0.9156
			47.37	0.8878
			48.25	0.8530
			49.12	0.8107
			50.00	0.7608
			50.87	0.7036
			51.75	0.6402
			52.62	0.5721
			53.50	0.5016
			54.37	0.4310
			55.25	0.3628
			56.12	0.2991
			57.00	0.2416
			57.87	0.1914
			58.75	0.1488
			59.62	0.1137
			60.50	0.0855
			61.37	0.0634
			62.25	0.0464
			63.12	0.0336
			64.00	0.0241
			64.87	0.0172
			65.75	0.0121
			66.62	0.0085

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 22.50
ALPHA=0.05 BETA=0.10 NPRIME= 15
ZA= 3.92 XABAR= 53.54

N	TL	TU	LOT XBAR	PROB ACC
1	-34.75	141.83	41.25	0.9867
2	25.10	189.06	42.12	0.9818
3	84.95	236.30	43.00	0.9751
4	144.80	283.53	43.87	0.9661
5	204.64	330.77	44.75	0.9541
6	264.49	378.00	45.62	0.9382
7	324.34	425.24	46.50	0.9175
8	384.19	472.47	47.37	0.8910
9	444.03	519.71	48.25	0.8576
10	503.88	566.94	49.12	0.8165
11	563.73	614.18	50.00	0.7673
12	623.58	661.41	50.87	0.7101
13	683.42	708.65	51.75	0.6456
14	743.27	755.88	52.62	0.5761
15	803.12	803.12	53.50	0.5034
			54.37	0.4307
			55.25	0.3606
			56.12	0.2957
			57.00	0.2377
			57.87	0.1878
			58.75	0.1459
			59.62	0.1118
			60.50	0.0847
			61.37	0.0635
			62.25	0.0472
			63.12	0.0349
			64.00	0.0256
			64.87	0.0187
			65.75	0.0137
			66.62	0.0099

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 22.50
ALPHA=0.05 BETA=0.10 NPRIME= 11
ZA= 5.08 XABAR= 53.50

N	TL	TU	LOT XBAR	PROB ACC
1	-60.79	167.78	41.25	0.9881
2	4.14	209.85	42.12	0.9831
3	69.07	251.92	43.00	0.9763
4	133.99	293.99	43.87	0.9669
5	198.92	336.06	44.75	0.9542
6	263.85	378.13	45.62	0.9375
7	328.77	420.20	46.50	0.9156
8	393.70	462.27	47.37	0.8877
9	458.63	504.34	48.25	0.8528
10	523.55	546.41	49.12	0.8104
11	588.48	588.48	50.00	0.7603
			50.87	0.7029
			51.75	0.6391
			52.62	0.5707
			53.50	0.4999
			54.37	0.4290
			55.25	0.3606
			56.12	0.2969
			57.00	0.2395
			57.87	0.1894
			58.75	0.1471
			59.62	0.1122
			60.50	0.0843
			61.37	0.0625
			62.25	0.0457
			63.12	0.0331
			64.00	0.0237
			64.87	0.0166
			65.75	0.0119
			66.62	0.0083

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 30.00
ALPHA=0.05 BETA=0.10 NPRIME= 27
ZA= 5.24 XBEAR= 53.55

N	TL	TU	LOT XBAR	PROB ACC
1	-103.66	210.76	41.25	0.9864
2	-44.06	258.26	42.12	0.9815
3	15.53	305.77	43.00	0.9748
4	75.13	353.27	43.87	0.9659
5	134.73	400.77	44.75	0.9540
6	194.32	448.28	45.62	0.9384
7	253.92	495.78	46.50	0.9180
8	313.52	543.28	47.37	0.8918
9	373.11	590.79	48.25	0.8588
10	432.71	638.29	49.12	0.8181
11	492.31	685.80	50.00	0.7691
12	551.90	733.30	50.87	0.7120
13	611.50	780.80	51.75	0.6475
14	671.10	828.31	52.62	0.5774
15	730.69	875.81	53.50	0.5042
16	790.29	923.31	54.37	0.4309
17	849.89	970.82	55.25	0.3603
18	909.48	1018.32	56.12	0.2951
19	969.08	1065.82	57.00	0.2370
20	1028.68	1113.33	57.87	0.1871
21	1088.27	1160.83	58.75	0.1454
22	1147.87	1208.33	59.62	0.1116
23	1207.47	1255.84	60.50	0.0847
24	1267.06	1303.34	61.37	0.0637
25	1326.66	1350.85	62.25	0.0476
26	1386.26	1398.35	63.12	0.0353
27	1445.85	1445.85	64.00	0.0261
			64.87	0.0192
			65.75	0.0141
			66.62	0.0103

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 30.00
ALPHA=0.05 BETA=0.10 NPRIME= 19
ZA= 6.43 XBEAR= 53.54

N	TL	TU	LOT XBAR	PROB ACC
1	-139.28	246.36	41.25	0.9875
2	-75.03	289.18	42.12	0.9825
3	-10.78	332.01	43.00	0.9757
4	53.47	374.83	43.87	0.9665
5	117.72	417.66	44.75	0.9541
6	181.97	460.48	45.62	0.9378
7	246.22	503.31	46.50	0.9165
8	310.47	546.13	47.37	0.8893
9	374.72	588.96	48.25	0.8553
10	438.97	631.78	49.12	0.8136
11	503.22	674.61	50.00	0.7641
12	567.47	717.44	50.87	0.7070
13	631.72	760.26	51.75	0.6433
14	695.97	803.09	52.62	0.5745
15	760.21	845.91	53.50	0.5031
16	824.46	888.74	54.37	0.4315
17	888.71	931.56	55.25	0.3624
18	952.96	974.39	56.12	0.2982
19	1017.21	1017.21	57.00	0.2405
			57.87	0.1903
			58.75	0.1480
			59.62	0.1133
			60.50	0.0856
			61.37	0.0638
			62.25	0.0471
			63.12	0.0344
			64.00	0.0250
			64.87	0.0180
			65.75	0.0129
			66.62	0.0092

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 15.00
ALPHA=0.01 BETA=0.01 NPRIME= 22
ZA= 4.36 XABAR= 52.50

N	TL	TU	LOT XBAR	PROB ACC
1	-13.14	118.15	44.25	0.9933
2	42.48	167.52	44.80	0.9905
3	98.11	216.89	45.35	0.9867
4	153.74	266.27	45.90	0.9815
5	209.36	315.64	46.45	0.9741
6	264.99	365.02	47.00	0.9641
7	320.61	414.39	47.55	0.9504
8	376.24	463.77	48.10	0.9320
9	431.87	513.14	48.65	0.9075
10	487.49	562.52	49.20	0.8757
11	543.12	611.89	49.75	0.8352
12	598.75	661.27	50.30	0.7850
13	654.37	710.64	50.85	0.7250
14	710.00	760.02	51.40	0.6560
15	765.63	809.39	51.95	0.5799
16	821.25	858.77	52.50	0.5001
17	876.88	908.14	53.05	0.4202
18	932.51	957.51	53.60	0.3442
19	988.13	1006.89	54.15	0.2752
20	1043.76	1056.26	54.70	0.2151
21	1099.39	1105.64	55.25	0.1650
22	1155.01	1155.01	55.80	0.1244
			56.35	0.0925
			56.90	0.0681
			57.45	0.0496
			58.00	0.0359
			58.55	0.0259
			59.10	0.0186
			59.65	0.0133
			60.20	0.0095

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 15.00
ALPHA=0.01 BETA=0.01 NPRIME= 15
ZA= 5.22 XABAR= 52.50

N	TL	TU	LOT XBAR	PROB ACC
1	-25.83	130.83	44.25	0.9936
2	32.26	177.74	44.80	0.9908
3	90.36	224.64	45.35	0.9869
4	148.46	271.55	45.90	0.9813
5	206.55	318.46	46.45	0.9735
6	264.65	365.36	47.00	0.9627
7	322.74	412.27	47.55	0.9482
8	380.84	459.17	48.10	0.9283
9	438.93	506.08	48.65	0.9024
10	497.03	552.98	49.20	0.8691
11	555.13	599.89	49.75	0.8274
12	613.22	646.79	50.30	0.7768
13	671.32	693.70	50.85	0.7172
14	729.41	740.60	51.40	0.6499
15	787.51	787.51	51.95	0.5766
			52.50	0.5001
			53.05	0.4236
			53.60	0.3503
			54.15	0.2829
			54.70	0.2234
			55.25	0.1727
			55.80	0.1310
			56.35	0.0977
			56.90	0.0718
			57.45	0.0521
			58.00	0.0374
			58.55	0.0266
			59.10	0.0188
			59.65	0.0131
			60.20	0.0092

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XAQL= 45.00 XLTPD= 60.00 SIGMA= 22.50
ALPHA=0.01 BETA=0.01 NPRIME= 34
ZA= 7.76 XBAR= 52.50

N	TL	TU	LOT XBAR	PROB ACC
1	-122.13	227.13	44.25	0.9938
2	-64.34	274.34	44.58	0.9923
3	-6.55	321.55	44.91	0.9905
4	51.24	368.76	45.24	0.9883
5	109.04	415.97	45.57	0.9855
6	166.83	463.17	45.90	0.9821
7	224.62	510.38	46.23	0.9780
8	282.41	557.59	46.56	0.9730
9	340.20	604.80	46.89	0.9669
10	398.00	652.01	47.22	0.9595
11	455.79	699.21	47.55	0.9507
12	513.58	746.42	47.88	0.9401
13	571.37	793.63	48.21	0.9274
14	629.16	840.84	48.54	0.9125
15	686.96	888.05	48.87	0.8950
16	744.75	935.26	49.20	0.8746
17	802.54	982.46	49.53	0.8511
18	860.33	1029.67	49.86	0.8242
19	918.12	1076.88	50.19	0.7939
20	975.92	1124.09	50.52	0.7601
21	1033.71	1171.30	50.85	0.7230
22	1091.50	1218.51	51.18	0.6827
23	1149.29	1265.71	51.51	0.6397
24	1207.08	1312.92	51.84	0.5944
25	1264.88	1360.13	52.17	0.5476
26	1322.67	1407.34	52.50	0.5000
27	1380.46	1454.55	52.83	0.4524
28	1438.25	1501.75	53.16	0.4056
29	1496.04	1548.96	53.49	0.3604
30	1553.84	1596.17	53.82	0.3173
31	1611.63	1643.38	54.15	0.2770
32	1669.42	1690.59	54.48	0.2399
33	1727.21	1737.80	54.81	0.2061
34	1785.00	1785.00	55.14	0.1758
			55.47	0.1489
			55.80	0.1254
			56.13	0.1050
			56.46	0.0875
			56.79	0.0726
			57.12	0.0599
			57.45	0.0493
			57.78	0.0405
			58.11	0.0331
			58.44	0.0270
			58.77	0.0220
			59.10	0.0179
			59.43	0.0145
			59.76	0.0117
			60.09	0.0095
			60.42	0.0077

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

XACL= 45.00 XLTPD= 60.00 SIGMA= 22.50
ALPHA=0.01 BETA=0.01 NPRIME= 48
ZA= 6.89 XBBAR= 52.53

N	TL	TU	LOT XBAR	PROP ACC
1	-102.56	207.61	44.25	0.9938
2	-46.73	256.84	44.58	0.9924
3	9.10	306.07	44.91	0.9906
4	64.93	355.30	45.24	0.9885
5	120.75	404.53	45.57	0.9859
6	176.58	453.76	45.90	0.9827
7	232.41	502.98	46.23	0.9789
8	288.24	552.21	46.56	0.9742
9	344.07	601.44	46.89	0.9685
10	399.89	650.67	47.22	0.9616
11	455.72	699.90	47.55	0.9533
12	511.55	749.13	47.88	0.9434
13	567.38	798.36	48.21	0.9315
14	623.21	847.58	48.54	0.9174
15	679.03	896.81	48.87	0.9008
16	734.86	946.04	49.20	0.8812
17	790.69	995.27	49.53	0.8585
18	846.52	1044.50	49.86	0.8324
19	902.34	1093.73	50.19	0.8026
20	958.17	1142.96	50.52	0.7692
21	1014.00	1192.18	50.85	0.7320
22	1069.83	1241.41	51.18	0.6914
23	1125.66	1290.64	51.51	0.6477
24	1181.48	1339.87	51.84	0.6014
25	1237.31	1389.10	52.17	0.5533
26	1293.14	1438.33	52.50	0.5042
27	1348.97	1487.55	52.83	0.4550
28	1404.80	1536.78	53.16	0.4067
29	1460.62	1586.01	53.49	0.3601
30	1516.45	1635.24	53.82	0.3159
31	1572.28	1684.47	54.15	0.2747
32	1628.11	1733.70	54.48	0.2369
33	1683.94	1782.93	54.81	0.2028
34	1739.76	1832.15	55.14	0.1724
35	1795.59	1881.38	55.47	0.1457
36	1851.42	1930.61	55.80	0.1224
37	1907.25	1979.84	56.13	0.1024
38	1963.07	2029.07	56.46	0.0852
39	2018.90	2078.30	56.79	0.0707
40	2074.73	2127.53	57.12	0.0585
41	2130.56	2176.75	57.45	0.0482
42	2186.39	2225.98	57.78	0.0397
43	2242.21	2275.21	58.11	0.0326
44	2298.04	2324.44	58.44	0.0267
45	2353.87	2373.67	58.77	0.0219
46	2409.70	2422.90	59.10	0.0179
47	2465.53	2472.13	59.43	0.0146
48	2521.35	2521.35	59.76	0.0119
			60.09	0.0097
			60.42	0.0079

SEQUENTIAL NON-PROBABILITY RATIO SAMPLING PLAN

YAQL= 45.00 XLTID= 60.00 SIGMA= 30.00
ALPHA=0.01 BETA=0.01 NPRIME= 60
ZA= 10.21 XALAR= 52.50

N	TL	TU	LOT MEAN	PROB ACC
1	-253.89	358.89	44.25	0.9937
2	-196.20	406.20	44.44	0.9929
3	-138.51	453.51	44.64	0.9920
4	-80.81	500.81	44.83	0.9909
5	-23.12	548.12	45.03	0.9897
6	34.57	595.43	45.22	0.9884
7	92.27	642.73	45.41	0.9868
8	149.96	690.04	45.61	0.9851
9	207.65	737.35	45.80	0.9832
10	265.35	784.66	46.00	0.9810
11	323.04	831.96	46.19	0.9786
12	380.73	879.27	46.39	0.9759
13	438.43	926.58	46.58	0.9728
14	496.12	973.88	46.77	0.9694
15	553.81	1021.19	46.97	0.9656
16	611.51	1068.50	47.16	0.9613
17	669.20	1115.80	47.36	0.9565
18	726.89	1163.11	47.55	0.9512
19	784.59	1210.42	47.74	0.9453
20	842.28	1257.73	47.94	0.9387
21	899.97	1305.03	48.13	0.9314
22	957.66	1352.34	48.33	0.9234
23	1015.36	1399.65	48.52	0.9145
24	1073.05	1446.95	48.71	0.9048
25	1130.74	1494.26	48.91	0.8941
26	1188.44	1541.57	49.10	0.8823
27	1246.13	1588.87	49.30	0.8696
28	1303.82	1636.18	49.49	0.8557
29	1361.52	1683.49	49.69	0.8406
30	1419.21	1730.80	49.88	0.8244
31	1476.90	1778.10	50.07	0.8069
32	1534.60	1825.41	50.27	0.7882
33	1592.29	1872.72	50.46	0.7683
34	1649.98	1920.02	50.66	0.7471
35	1707.[illegible]	[illegible]	50.85	0.7248
36	1765.37	2014.64	51.04	0.7014
37	1823.06	2061.94	51.24	0.6769
38	1880.76	2109.25	51.43	0.6514
39	1938.45	2156.56	51.63	0.6251
40	1996.14	2203.87	51.82	0.5981
41	2053.84	2251.17	52.01	0.5705
42	2111.53	2298.48	52.21	0.5425
43	2169.22	2345.79	52.40	0.5142
44	2226.92	2393.09	52.60	0.4858
45	2284.61	2440.40	52.79	0.4576
46	2342.30	2487.71	52.99	0.4295
47	2399.99	2535.01	53.18	0.4019
48	2457.69	2582.32	53.37	0.3749
49	2515.38	2629.63	53.57	0.3486
50	2573.07	2676.94	53.76	0.3232
51	2630.77	2724.24	53.96	0.2987
52	2688.46	2771.55	54.15	0.2752
53	2746.15	2818.86	54.34	0.2529
54	2803.85	2866.16	54.54	0.2318
55	2861.54	2913.47	54.73	0.2118
56	2919.23	2960.78	54.93	0.1931
57	2976.93	3008.08	55.12	0.1757
58	3034.62	3055.39	55.31	0.1594
59	3092.31	3102.70	55.51	0.1444
60	3150.01	3150.01	55.70	0.1305
			55.90	0.1177
			56.09	0.1060
			56.29	0.0952
			56.48	0.0855
			56.67	0.0766
			56.87	0.0686
			57.06	0.0613
			57.26	0.0547
			57.45	0.0488
			57.64	0.0435
			57.84	0.0387
			58.03	0.0344
			58.23	0.0306
			58.42	0.0272
			58.61	0.0241
			58.81	0.0214
			59.00	0.0190
			59.20	0.0168
			59.39	0.0149
			59.59	0.0132
			59.78	0.0116
			59.97	0.0103
			60.17	0.0091
			60.36	0.0080
			60.5[illegible]	[illegible]

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